Analysis of Disability Insurance Portfolios with Stochastic Interest Rates

Yu Xia (Lilian)

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Outline

- Introduction
- The Multi-state Transition Model
- Interest Rate Models
- Methodology of Moment Calculation and Risk Classification
- Numerical Results
- Conclusion
Review of Previous Research

- Haberman et al. (1997) introduced a general multi-state model for different types of long term care (LTC) insurance products.

- Parker (1997) introduced two cash flow approaches to valuate the average risk per policy by conditioning on the interest rate and the survivorship, respectively, for a traditional term life/endowment insurance portfolio under stochastic interest rate environment.
The Design of Our Product

- Policyholders are assumed healthy at their entry ages.
- The statue check is done semiannually.
- Semiannual TD (PD) benefits are provided
- Lump sum death benefit are provided
Why This Product?
Transition Assumptions

- The transition process follows Markov Chain

- Piecewise constant forces of transition:
  \[ \mu_{ij}^*(x + h) = \mu_{ij}^*(x) \text{ for any } 0 \leq h < 1. \]

- Further Assumptions concluded from the survey
  \[ \begin{align*}
  \mu_{10}(x) &= 0.1\mu_{01}(x) \\
  \mu_{03}(x) &= \mu_{13}(x) = \mu_{23}(x) \\
  \mu_{12}(x) &= \mu_{02}(x) \\
  2\mu_{02}(x) &= 3\mu_{01}(x)
  \end{align*} \]

- References: Haberman et al. (1997) and Rajnes (2010)
The service table is applied to model the transition intensities.

When the status check is done semiannually \((t_0 = 0.5)\), the transition probabilities can be approximated by

\[
0.5p^{ij}(x) = \begin{cases} 
\exp\{-0.5 \sum_{k \neq i} \mu^{ik}(x)\}, & i = j \\
0.5\mu^{ij}(x), & i \neq j
\end{cases}.
\]

References: Dickson et al. (2009)
The Interest Rate Models Used for Valuation Purpose

- Deterministic Interest Rate Model
- Binomial Tree Interest Rate Model
- AR(1) Model
Binomial Tree Interest Rate Model

Assumptions
- \( p = 0.5 \)
- The annual volatilities for the period \([k-1,k), \sigma_r(k)\), are time constant.
- References: Gaillardetz (2007)
The AR(1) Process

- Assume the annual forces of interest follow an AR(1) process:
  \[ \ln(1+\delta_k) - \ln(1+\delta_0) = \phi [\ln(1+\delta_{k-1}) - \ln(1+\delta_0)] + a_k, \]
  where \( a_k \sim N(0, \sigma^2_a) \).

- First calculate the expectation, variance and covariance terms of the sum of the forces of interest:
  \[ I(\delta_0, s) = \begin{cases} 
  \sum_{i=0}^{[s]-1} \ln(1+\delta_i) + \{s\} \ln(1+\delta_{[s]}) , & s \geq 1 \\
  s\ln(1+\delta_0) , & 0 \leq s < 1 
\end{cases} \]

- Calculate the expectation, variance and covariance terms of the discounted interest rate
The Cash Flow Calculation

- Two approaches of risk decomposition of the total riskiness
  - Conditioning on the interest rate
  - Conditioning on the transition process
First Approach

- Conditioning on the interest rate, the total riskiness per policy can be decomposed as:
  - The insurance risk
    \[
    \sum_{s=1}^{2n} \sum_{u=1}^{2n} \{ \mathbb{E}[V(\delta_0, 0.5s) V(\delta_0, 0.5u)] \text{Cov} [CF_{0.5s}, CF_{0.5u}] \} 
    \]
  - The investment risk
    \[
    \sum_{s=1}^{2n} \sum_{u=1}^{2n} \{ \text{Cov} [V(\delta_0, 0.5s), V(\delta_0, 0.5u)] \mathbb{E}[CF_{0.5s}] \mathbb{E}[CF_{0.5u}] \} 
    \]
Second Approach

- Conditioning on the transition assumption, the total riskiness per policy can be decomposed as:
  - The insurance risk
    \[ \sum_{s=1}^{2n} \sum_{u=1}^{2n} \left\{ E[V(\delta_0, 0.5s)] E[V(\delta_0, 0.5u)] Cov[CF_{0.5s}, CF_{0.5u}] \right\} \]
  - The investment risk
    \[ \sum_{s=1}^{2n} \sum_{u=1}^{2n} \left\{ Cov[V(\delta_0, 0.5s), V(\delta_0, 0.5u)] E[CF_{0.5s}CF_{0.5u}] \right\} \]
Single Policy Case

- The insurance risk and the investment risk across age groups for different terms of policies under the AR(1) interest rate model

- Sensitivity tests on the benefit payment amounts and the parameters in the AR(1) interest rate model

- Base Scenario: \( \phi = 0.9; \sigma_a = 0.01; \delta_0 = 0.06 \)
Single Policy Case

**Figure:** Insurance and Investment Risk of PVFBP vs Term Policy and Age under The AR(1) Interest Rate Model
($\phi = 0.9; \sigma_a = 0.01; \delta_0 = 0.06$)
Single Policy Case

**Figure:** Expectation and Variance of PVFBP vs Death Benefit $b_3$ for a 15-year Disability Insurance Policy under The AR(1) Interest Models ($b_1 = 1; b_2 = 2; \phi = 0.9; \sigma_a = 0.01; \delta_0 = 0.06$)
Figure: Insurance and Investment Risk vs $\phi$ and Age for a 15-year Disability Insurance Policy under The AR(1) Interest Rate Models ($\sigma_a = 0.01; \delta_0 = 0.06$)
Figure: Insurance and Investment Risk vs $\delta_0$ and Age for a 15-year Disability Insurance Policy under The AR(1) Interest Rate Models ($\sigma_a = 0.01; \phi = 0.9$)
Single Policy Case

Figure: Insurance and Investment Risk vs $\sigma_a$ and Age for a 15-year Disability Insurance Policy under The AR(1) Interest Rate Models ($\delta_0 = 0.06; \phi = 0.9$)
Portfolio Case

- Three interest rate models
  - Deterministic interest rate model: $\delta_0 = 0.06$
  - Binomial tree interest rate model: $r = \delta_0 = 0.06; \sigma_r(k) = 0.08$
  - AR(1) interest rate model: $\phi = 0.9; \sigma_a = 0.01; \delta_0 = 0.06$

- Risk analysis of a general insurance portfolio

- Comparisons among our new product and the traditional term life/disability products
Table: The Policy Information of The General Insurance Portfolio

<table>
<thead>
<tr>
<th>Group No.</th>
<th>No. of Policies</th>
<th>Age</th>
<th>Term</th>
<th>TD Benefit</th>
<th>PD Benefit</th>
<th>Death Benefit</th>
</tr>
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<tbody>
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<td>1</td>
<td>80</td>
<td>35</td>
<td>5</td>
<td>1,000</td>
<td>2,000</td>
<td>30,000</td>
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<tr>
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<td>10</td>
<td>2,000</td>
<td>4,000</td>
<td>60,000</td>
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<td>35</td>
<td>15</td>
<td>3,000</td>
<td>6,000</td>
<td>90,000</td>
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<tr>
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<td>100</td>
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<tr>
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<td>60</td>
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<td>500</td>
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<td>40</td>
<td>15</td>
<td>5,000</td>
<td>10,000</td>
<td>150,000</td>
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<tr>
<td>8</td>
<td>180</td>
<td>40</td>
<td>20</td>
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<td>9</td>
<td>400</td>
<td>45</td>
<td>5</td>
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<td>90,000</td>
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<tr>
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<td>45</td>
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<tr>
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</tbody>
</table>
Portfolio Case

Table: The Risk Decomposition for The Non-homogeneous Long-term Disability Insurance Portfolios under Three interest Models

<table>
<thead>
<tr>
<th>No. of Policies</th>
<th>Deterministic</th>
<th>Binomial</th>
<th>AR(1)</th>
<th>Binomial</th>
<th>AR(1)</th>
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<tbody>
<tr>
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</tr>
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<td>44,800,000</td>
<td>9.301</td>
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<td>58,668</td>
</tr>
<tr>
<td>Infinity</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10,136.2</td>
<td>58,668</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>No. of Policies</th>
<th>Deterministic</th>
<th>Binomial</th>
<th>AR(1)</th>
<th>Binomial</th>
<th>AR(1)</th>
</tr>
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<tbody>
<tr>
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<td>93,014</td>
<td>93,011</td>
<td>93,856</td>
<td>10,470.1</td>
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<tr>
<td>44,800</td>
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<tr>
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<td>93.01</td>
<td>93.01</td>
<td>93.9</td>
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<td>58,670</td>
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<tr>
<td>44,800,000</td>
<td>9.301</td>
<td>9.3</td>
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<td>58,668</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>10,136.2</td>
<td>58,668</td>
</tr>
</tbody>
</table>
Table: The Risk Decomposition Comparison for The Term Life Insurance Portfolio, Disability Only Portfolio and The Long-term Disability Insurance Portfolios under Three Interest Rate Models

<table>
<thead>
<tr>
<th></th>
<th>No. of Policies</th>
<th>Benefits</th>
<th>Deterministic</th>
<th>Binomial</th>
<th>AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio 1</td>
<td>4,480</td>
<td>Death</td>
<td>335,825,056</td>
<td>335,815,917</td>
<td>338,391,200</td>
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<tr>
<td>Portfolio 2</td>
<td>4,480</td>
<td>Disabilities</td>
<td>91,057,971</td>
<td>91,052,237</td>
<td>92,406,720</td>
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<tr>
<td><strong>Sum of 1+2</strong></td>
<td>8,960</td>
<td>N/A</td>
<td>426,883,027</td>
<td>426,868,154</td>
<td>430,797,920</td>
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<tr>
<td><strong>Our Portfolio</strong></td>
<td>4,480</td>
<td>Both</td>
<td>416,704,333</td>
<td>416,690,042</td>
<td>420,475,462</td>
</tr>
</tbody>
</table>

Total Investment Risk

<table>
<thead>
<tr>
<th></th>
<th>No. of Policies</th>
<th>Benefits</th>
<th>Deterministic</th>
<th>Binomial</th>
<th>AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio 1</td>
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<td>Death</td>
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<td>21,922,374</td>
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<td>160,965,226</td>
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<tr>
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<td>Both</td>
<td>0</td>
<td>45,409,952</td>
<td>262,831,341</td>
</tr>
</tbody>
</table>
Conclusion

- The investment risk per policy is extremely large with very long-term policies under the binomial tree model.

- The investment risk is sensitive to $\sigma_a$ at large values of $\sigma_a$ and senior ages under the AR(1) interest rate model. It is also extremely sensitive to $\phi$ when $\phi$ is close to 1.

- The new long-term disability product provides accuracy in valuation.
Future Work

- With insurance data available, the semi-Markov chain model which considers the effect of the duration can be applied to model the transition process.

- Surplus calculation

- Short-term cash flow projections with appropriate certificate growth and lapse assumption

- Allowance of different statuses at the time of entry
Thank you for listening!