Dynamic Population Structure with Stochastic Mortality and Fertility Rates

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Objectives

- Propose a population structure model based on the Leslie matrix, validate the model with U.S. population data.
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- Predict future population structure and explore the impact on the future labour force stability
- Investigate the impact of a DC pension system on the labor force stability, based on the predicted population structure
Leslie Matrix

- \( n_x \): Population size of each age group \( x \)
- \( s_x \): Survival probability for \( x \), \((n_{x+1})_{t+1} = s_x(n_x)_t\)
- \( f_x \): Total birth rate among those who are at age group \( x \)

At time \( t + 1 \), the population of the new born is

\[
(n_0)_{t+1} = \sum_{x=0}^{\omega-1} f_x(n_x)_t
\]

\[
\begin{pmatrix}
n_0 \\
n_1 \\
n_2 \\
\vdots \\
n_{\omega-1}
\end{pmatrix}_{t+1} =
\begin{pmatrix}
f_0 & f_1 & f_2 & \cdots & f_{\omega-2} & f_{\omega-1} \\
s_0 & 0 & 0 & \cdots & 0 & 0 \\
0 & s_1 & 0 & \cdots & 0 & 0 \\
0 & 0 & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & \ddots & S_{\omega-2} & 0 \\
0 & 0 & 0 & \cdots & 0 & s_{\omega-1}
\end{pmatrix}
\begin{pmatrix}
n_0 \\
n_1 \\
n_2 \\
\vdots \\
n_{\omega-1}
\end{pmatrix}_t
\]

This can be written as, \( \mathbf{n}_{t+1} = \mathbf{Ln}_t \).
Leslie Matrix

- Historical data not satisfy the stable growth rate assumption
Leslie Matrix

- Historical data not satisfy the stable growth rate assumption
- Leslie Model modification

\[ n_{t+1} = Ln_t \]

\[ \downarrow \]

\[ n_{t+1} = L_t n_t \]
Leslie Matrix

- Historical data not satisfy the stable growth rate assumption
- Leslie Model modification

\[ n_{t+1} = L_n t \]

\[ \Downarrow \]

\[ n_{t+1} = L_t n_t \]

- Population is divided into five-year age groups and projected every five years
Projected Population Structure

Figure: Projected Population Size in 1955 and 1995
Projected Population Structure

Figure: Projected and Observed Population Size in 1995
The Impact of Immigration Population

\( n_x \) is the current population size for age group \( x \);
\( n \) is the total population size for all ages;
\( t_x \) is the immigration population for age group \( x \);
\( t \) is the total immigration population size.

**Assumption**: Immigration population has the same age distribution as the original population

\[
\frac{t_x}{t} = \frac{n_x}{n}
\]

After immigration, the population percentage for age group \( x \) is,

\[
\frac{n_x + t_x}{n + t} = \frac{n_x + n_xt/n}{n + t} = \frac{(n_x n + n_x t)/n}{n + t} = \frac{n_x}{n}
\]
Projected Population Structure

**Figure:** Projected and Observed Population Percentage in 1995
Lee-Carter Model

\[
\log m_{x,t} = a_x + b_x k_t + \epsilon_{x,t}
\]

\[
k_{t+1} = k_t + c + e_t, \quad e_t \sim N(0, \sigma^2)
\]

\(m_{x,t}\) describes the observed central death rate at age \(x\) in year \(t\); 
\(a_x\) describes the age pattern of mortality averaged over time; 
\(b_x\) describes the deviations from the averaged pattern when \(k_t\) varies; 
\(k_t\) describes the variation in the level of mortality over time.
Fertility Forecasting Using Lee-Carter Model

Figure: Fertility Rates by Age of Mother from 1976-2006

(a) Before log Transformation 
(b) After log Transformation

Figure: Fertility Rates by Age of Mother from 1976-2006
Fertility Forecasting (Comparison)

Figure: Projected and Observed Population Percentage in 1995
Fertility Forecasting (Comparison)

Figure: Projected and Observed Population Percentage in 2005
Future Population Structure Prediction

Figure: Predicted Population Percentage in future years.
## Model Stability Testing

<table>
<thead>
<tr>
<th>Age</th>
<th>0-4</th>
<th>5-9</th>
<th>10-14</th>
<th>15-19</th>
<th>20-24</th>
<th>25-29</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.0635</td>
<td>0.0622</td>
<td>0.0600</td>
<td>0.0588</td>
<td>0.0576</td>
<td>0.0556</td>
</tr>
<tr>
<td>sd</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td>Age</td>
<td>30-34</td>
<td>35-39</td>
<td>40-44</td>
<td>45-49</td>
<td>50-54</td>
<td>55-59</td>
</tr>
<tr>
<td>mean</td>
<td>0.0597</td>
<td>0.0593</td>
<td>0.0597</td>
<td>0.0560</td>
<td>0.0571</td>
<td>0.0585</td>
</tr>
<tr>
<td>sd</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>Age</td>
<td>60-64</td>
<td>65-69</td>
<td>70-74</td>
<td>75-79</td>
<td>80-84</td>
<td>85-89</td>
</tr>
<tr>
<td>mean</td>
<td>0.0634</td>
<td>0.0603</td>
<td>0.0521</td>
<td>0.0432</td>
<td>0.0309</td>
<td>0.0216</td>
</tr>
<tr>
<td>sd</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0004</td>
</tr>
<tr>
<td>Age</td>
<td>90-94</td>
<td>95-99</td>
<td>100-104</td>
<td>105-109</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.0141</td>
<td>0.0059</td>
<td>0.0006</td>
<td>0.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sd</td>
<td>0.0005</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table: Population Percentage Structure in 2025 (5000 simulation)
Dependency Ratio Prediction (Fixed Retirement Age)

\[
Aged \ Denpendency \ Ratio = \frac{\# \ of \ people \ aged \ 65+}{\# \ of \ people \ aged \ 15 - 64}
\]

Figure: Aged Dependency Ratio when Fixed Retirement Age is 65

The prediction is stable and the aged dependency ratio is expected to be 38\% by 2025, closed to 34\% that estimated by Turner and Watanabe (1995).
Dependency Ratio Under DC Pension System

Assumed that participants retire when their future pension income exceed $\frac{2}{3}$ of current salary.

$$Retire\ Age = \min\{x : \frac{Wealth_x}{\bar{a}_x} \geq \frac{2}{3} \}$$

$$Aged\ Dependency\ Ratio = \frac{\#\ retired\ population}{\#\ working\ population}$$

<table>
<thead>
<tr>
<th>Asset</th>
<th>100% bond</th>
<th>50% bond &amp; 50% equity</th>
<th>100% equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>DR Mean</td>
<td>0.338</td>
<td>0.5955</td>
<td>0.7242</td>
</tr>
<tr>
<td>DR sd</td>
<td>0.1329</td>
<td>0.2136</td>
<td>0.2714</td>
</tr>
<tr>
<td>Equivalent Retire Age</td>
<td>64.33</td>
<td>64.19</td>
<td>53.09</td>
</tr>
</tbody>
</table>

Table: Simulated mean, sd of aged dependency ratio in 2025 and the equivalent retirement age
Conclusions and Limitations

- The financial security system is becoming a major concern as the baby boom generation approach their retirement.
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- DC pension system has a strong impact on the labour force stability of the population.
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- The financial security system is becoming a major concern as the baby boom generation approach their retirement.
- DC pension system has a strong impact on the labour force stability of the population.
- The fertility forecasting using Lee-Carter model has its limitation as it depends a lot on the selection of an appropriate estimation period.
Thank you!