

You should attempt as many questions on the quiz as you can. You are not expected to solve every problem. What is important is evidence of your reasoning. Justify your solutions. If you get stuck in a problem, give a partial solution. Be patient; there is no time limit.

1. Solve each of the following equations for  $x$ :

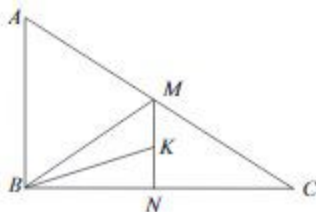
(a)  $3^{x+4} = 9^x$     (b)  $8^{x^2-2} = 4^{2-x}$

2. The lengths of the sides of a rectangular solid are indicated in the figure to the right. Find the length of the diagonal  $AB$ .



3. Prove that  $n^3 + 3n^2 + 2n$ , where  $n$  is a positive integer, is always divisible by 6.
4. Pipe A can fill a swimming pool in 20 hours. Pipe B can fill the pool in 25 hours. Drain pipe C can empty the pool in 30 hours. How long would it take to fill an empty pool if both pipes A and B are operating and the drain is open at the same time?

5. Triangle  $ABC$  in the figure to the right has a right angle at  $B$  with side  $AC = 5$  m and side  $BC = 4$  m.  $BM$  is a median of triangle  $ABC$ .  $MN$  is a median of triangle  $MBC$ .  $BK$  is a median of triangle  $BMN$ . Find the area of triangle  $BKM$ .



6. In a game, there are three piles of stones. The first pile has 22 stones, the second has 14 stones, and the third has 12 stones. At each turn, you may double the number of stones in any pile by transferring stones to it from one other pile. The game ends when all three piles have the same number of stones. Find the minimum number of turns to end the game.
7. Determine whether it is possible to find positive integers  $m$  and  $n$  so that  $m(m+1) = n(n+2)$ .
8. Is it possible to place 26 points inside a rectangle that is 20 cm by 15 cm so that the distance between every pair of points is greater than 5 cm?
9. Your task is to house 100 pigeons in cages so that each cage contains at least one pigeon and no cage contains the same number of pigeons. Find the maximum number of cages that you can use.
10. Find all pairs of integers  $x$  and  $y$  that satisfy the equation  $x^2 - y^2 = 99$ .