LCP 2: MOTION AND THE PENDULUM

Without the pendulum there would have been no Principia. (R.S. Westfall)

All of physics comes from Galileo's inclined plane. (Anon)

Fig. 1: Dome of the Cathedral of Pisa with the "Lamp of Galileo"

While watching a chandelier swing back and forth at the Cathedral of Pisa in 1583, Galileo noticed something curious. Galileo noticed that the time period to swing through one complete cycle is independent of the amplitude through which it swings…..He timed the swings with his pulse, the only timing device at hand. (IL 1)

IL 1 ***

IL 2 ***

Fig. 2: Foucault's Pendulum, the Pantheon, Paris

That was when I saw the pendulum. The sphere, hanging from a long wire set into the ceiling of the choir, swayed back and forth with isochronal majesty. I knew that the period was governed by the square root of the length of the wire and by π, that number …which binds the circumference and diameter of all possible circles…(Umberto Eco, taken from his book "Foucault's Pendulum).

IL 3 **** Description of our pendulum book just published

IL 4 *** Galileo's original description of his inclined plane experiment

IL 5 **** Summary of the book on the Pendulum (scroll down to "The Ubiquitous Pendulum")

IL 6 **** A good brief history of the pendulum with nice pictures

IL 7 **** An excellent video replicating Galileo's experiment

IL 8 ** A brief biography of Galileo and some of his work

IL 9 ** A summary of the International Pendulum Project (IPP)

© 2007, Arthur Stinner

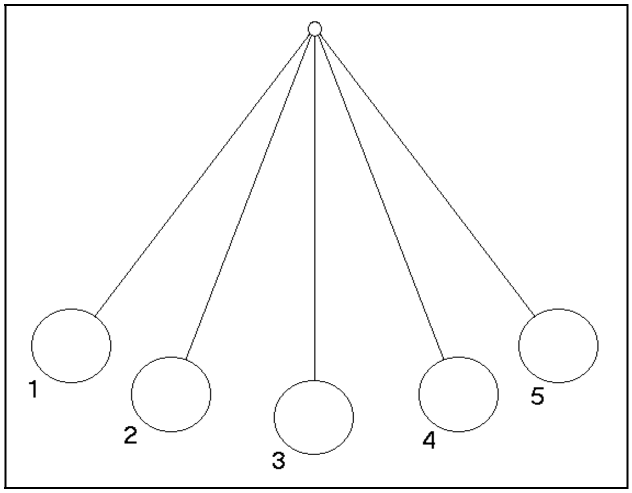


Fig. 3: The Pendulum at Different Stages of Its Oscillation I

THE MAIN IDEA

This LCP is guided by the history of the pendulum, based on the work of six philosophers and scientists on motion: Aristotle, Nicole Oresme, Galileo, Christian Huygens, Newton, and Leon Foucault, from the fourth century BC to the middle of the 19th century. We will add a dramatization involving short conversations for each of them.

Each of the sections begins with a brief introduction, followed by a short dramatization for each philosopher/scientist and concludes with a list of suggestions for discussion, questions, problems and activities in the physics classroom. These dramatizations can be performed (read) in class by two students or by the instructor and a student in order to give the student a historical sense and dramatize and humanize the main ideas. It is suggested that students make these presentation. These exercises will force you to try to think like the scientist that you are pretending to be. (The “conversations” are also found in the Appendices).

You will notice that in following the history of the concepts about forces and motion by using the pendulum as the central idea, we will recapitulate (re-discover) the intuitive physics discussed in LCP 1. However, these ideas will be discussed in a richer context and on a more sophisticated level. Rich concepts, such as those found in Newton’s dynamics, must be mastered and ultimately understood in a deep sense, and not just superficially memorized from textbooks. If these important concepts and ideas are later presented in the same way as it was done the first time, learning also becomes boring and many of the ideas and concepts will become inert.

The pendulum did not only play a central role in the development of the kinematics and dynamics in the seventeenth century, but was also a research instrument in the 18th and 19th centuries and is still used today in the study of chaos theory.

It is recommended that before the questions, problem and experimental sections are attempted, for each case, students present the brief conversation to the whole class, followed by a discussion.

© 2007, Arthur Stinner

The ubiquitous (look up the meaning of the word) pendulum, together with the versatile inclined plane, played a central role in the development of the kinematics and dynamics in the seventeenth century. In many of the key problems of Galileo these simple devices were connected in creative ways to study motion, without considering the forces involved (kinematics). Textbooks tell students that Galileo "diluted gravity" and extrapolated to free fall in an attempt to understand free fall, or what Aristotle called "natural motion". However, textbooks generally don't mention that Galileo also studied free fall directly, using an ingenious method of timing the fall with a pendulum. The "interrupted" swing of the pendulum anticipated the principle of the conservation of mechanical energy. Studying the pendulum, Galileo thought that an arc of a circle represented the "least time" path of an object in a vertical plane. He was almost right.

The history of the uses of the pendulum in the study of kinematics and dynamics contains everything required to teach the fundamentals of kinematics and dynamics on the elementary level.

A brief history of the pendulum should reveal the unfolding of most of the ideas and methods of the discipline of kinematics and dynamics that is central to elementary physics. The importance of the pendulum, however, goes beyond the 17^{th} century; we find the pendulum as a research instrument, right into the 20^{th} century.

THE DESCRIPTION OF THE CONTEXT

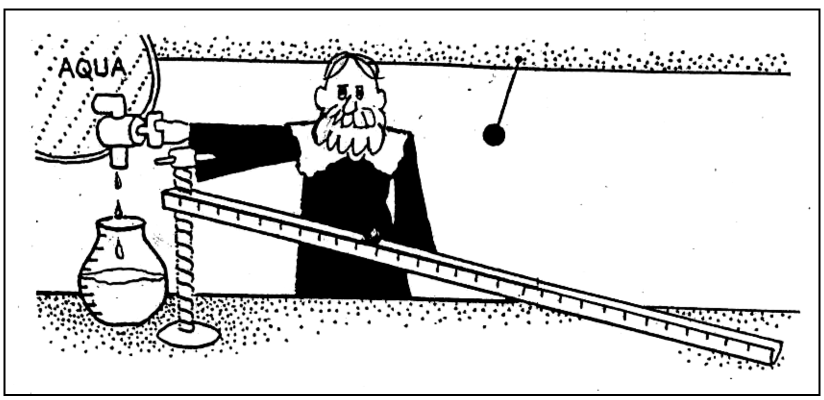


Fig. 4: Galileo's Inclined Plane

A Brief History of the Pendulum

The study of motion must begin with Aristotle's ideas. Aristotle (4^{th} century B.C.) was arguably the first to look at the physical world in a systematic, what we might today call a "rational" way. He argued for certain fundamental principles, which were based on naked-eye observations and partly on logic. But his explanations of motion cannot be considered scientific in the modern sense because he did not consider it necessary to check his principles by controlled experiments.

© 2007, Arthur Stinner

He insisted, however, that these principles were based on careful observation of moving objects in everyday life. He observed objects falling, smoke rising, carts moving as they were being pulled by oxen and spears and arrows moving through the air. In the heavens, he tried to explain the irregular motion of the planets. However, he thought that the physics of the heavens (celestial) motion was qualitatively different from the motion of objects on earth (terrestrial motion). He thought that his common sense account of motion in general was sufficient to explain all motion around us.

Aristotle had problems explaining projectile motion and therefore would have been puzzled by the motion of a pendulum. According to Aristotle, this kind of motion was a combination of violent and natural motion. The motion of a projectile (javelin) was difficult to explain using his physics: *natural motion*, like free fall, required no explanation, and *forced motion*, like the motion of a cart being pulled by a horse can be explained.

But what force keeps a projectile in motion or a pendulum oscillating after contact is lost with the projectile or the oscillating pendulum?

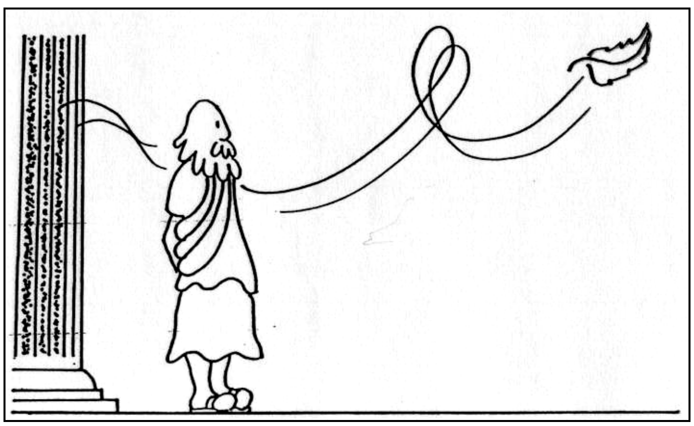


Fig. 5: Aristotle Observes Motion

IL 10 *** Motion according to Aristotle – with good animation.

The Middle Ages

One of the early scientists (natural philosophers) that openly challenged Aristotle's physics was the 14th century natural philosopher Nicole Oresme (1323 – 1382). He learned from his teacher, Buridan, that it is *impetus* that explains the motion of a projectile and a pendulum. Oresme proposed a "thought experiment" that is our first example of the motion of the pendulum being discussed by a natural philosopher. This thought experiment will be presented later.

IL 11 *** Motion according to Galileo – with animations.

IL 12 *** A brief history of the pendulum

IL 13 *** A brief biography of Galileo

© 2007, Arthur Stinner

Galileo and the Pendulum

The Italian physicist and mathematician Galileo Galilei (1564-1642) laid the foundations of kinematics, the study of motion without considering the forces that produce that motion. He was particularly interested in uniformly accelerated motion. Legend has it that he dropped two objects, one much heavier than the other from the Leaning Tower of Pisa when he was a young man. He most likely never checked his ideas about free fall by dropping objects from the Leaning Tower of Pisa. He apparently did not think this kind of demonstration was necessary.

Galileo recorded his thoughts and experiments in his book *The Two New Sciences* that was written in the last decade of his life. He wrote the text in Italian rather than in Latin as was generally done in those days, while under house arrest. Here he clearly described how, using an inclined plane, he "diluted" gravity and idealizing and extrapolating to free fall calculated (estimated) the distance a freely falling object falls in a given time. Today, we are more interested in the value of what we today call g, the acceleration due to gravity.

He was the first to show that trajectory motion is parabolic and that the period of the pendulum is fairly constant. He proved, and later confirmed by experiment that the period is proportional to square root of the length. He also argued that the arc of a circle represented the least time (brachistochrone) path of an object descending along a frictionless path in a vertical plane.

Galileo then turned to the question of what happens when a ball rolls horizontally, having rolled down an inclined plane. He argued that it should roll up another inclined plane and move to the same height. Later, he used an "interrupted" pendulum to demonstrate what we now call "conservation of mechanical energy". About 50 years later this idea was expressed and discussed by the German natural philosopher and mathematician Gottfried von Leibniz, in the form of kinetic energy, what he called "vis viva". In fact, the Dutch natural philosopher Huygens entered into a dispute with the German natural philosopher and mathematician Leibniz over whether kinetic energy or linear momentum were conserved in billiard ball collisions. The dispute was eventually resolved by realizing that in any elastic collision both must be conserved.

To illustrate this, Galileo observed that the motion of a pendulum will rise (almost) to its starting level even if a peg is used to the change the path. However, he went further and imagined that if a ball were to roll forever without resistance (a thought experiment) then the ball should circumnavigate the earth. This *thought experiment* is important in the development our understanding of motion. In fact, it anticipates Newton's first law of motion, generally referred to as the "law of inertia".

The 17th Century

Huygens

The Dutch physicist Christian Huygens (1629–1695) was second only to Newton as a natural philosopher and mathematician. He went beyond Galileo and used the pendulum to find the expression for "centrifugal" force as well as the modern formula for the period of a pendulum for small angles, namely that T = $2\pi\sqrt{l / g}$, and the first to write the mathematical statement for "centrifugal" acceleration as a = v^2 / R . He then used long and heavy pendula to determine the

© 2007, Arthur Stinner

value of gravitational acceleration. He later correlated latitude and the local value of g to test his ideas. Huygens was also the first to show (geometrically) that the path along which a pendulum would show isochronous motion was a cycloid and not the arc of a circle. From this background we can generate many experiments and problems that cover all those found in textbooks and beyond and in more interesting ways.

Huygens constructed the first pendulum clock to follow the path of a cycloid that kept fairly accurate time, based on his geometric proof that the cycloid and not the arc of a circle guaranteed an isochronous (equal time) motion. However, he failed to realize that the cycloid also represented the "least time" path of descent of a particle in a vertical plane. Later (1695), Johannes Bernoulli, Newton and others showed that the least time descent of an object in a vertical plane between two points is a cycloid.

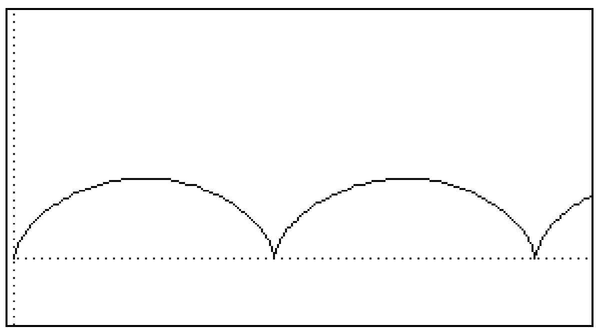


Fig. 6: The Cycloid

IL 14 ** Huygens biography

IL 15 *** Huygens biography

It was left to Newton (1642-1727), Leibniz (1646-1716) and Johannes Bernoulli (1667–1748) to lay the foundation of a new branch of the calculus, in order to solve problems such as the brachistochrone , or "least time" of descent between two points in a vertical plane. In the capable hands of the great Swiss mathematician Euler their approach then became a powerful method to solve minimum and maximum problems, called "variational calculus".

© 2007, Arthur Stinner

Fig. 7: A Sixteenth Century Demonstration: The "Quickest Descent"—A Discrepant Event

IL 16 *** Picture taken from "Galileo and the Pendulum".

IL 17 *** History of the brachistochrone problem

IL 18 *** IA of the brachistochrone problem

IL 19 *** IA of the brachistochrone problem

A simple apparatus can be built, using two wires, one straight and the other roughly shaped as a cycloid, with two ball bearings sliding down the wires. This is an example of a discrepant event that is sure to generate much discussion. The event is discrepant because the time it takes for the bead to slide down the longer path is actually shorter! The work of Robert Hooke (1640–1706), a contemporary of Newton, should be included in this historical presentation. Textbooks generally mention Hooke only in connection with his law of springs. Hook is often called "the British Leonardo". He was a polymath (a person who has extensive knowledge in many disciplines): scientist, inventor and arguably the greatest experimenter of the seventeenth century. He was the curator of the Royal Society and sometime friend of Newton. He used his law ($F = -kx$) to show that simple harmonic motion (SHM), like that of the pendulum, or an oscillating mass attached to a spring, arises when this law holds.

His scientific battles with Newton were legendary. When Newton became the president of the Royal Society in 1705, he expunged all vestiges of Hooke. Thus we have no likeness of him. We identify Robert Hooke by the famous drawing he made of a louse in his revolutionary "Micrographia" that he published at the age of 30. Discussing the confrontation between Newton and Hooke students quickly come to realize that science is a human endeavor and cannot be captured by a specifiable method. After Hooke's death in 1706, Newton was elected the presidency of the Royal Society. Unfortunately, Newton destroyed all vestiges of Hooke in the building, such as pictures and busts, so that today we only know what he looked like through descriptions.

© 2007, Arthur Stinner

Isaac Newton

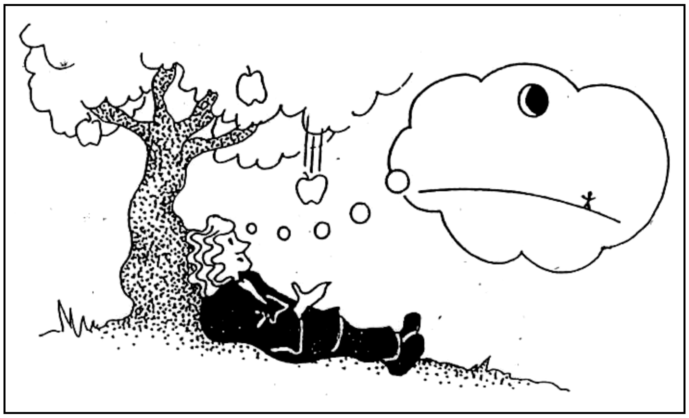


Fig. 8: Newton Contemplating Gravity

IL 20 *** A long biography of Newton

IL 21 ** A short biography of Newton

Isaac Newton (1642-1727) showed that the laws of motion applied to "things terrestrial" as well as to "things celestial". These laws and the inverse square law of gravity explained at once planetary motion, the motion of the tides, free fall, and the motion of the pendulum. We will see later how Newton used large wooden spheres as pendula for collision experiments, for testing "centrifugal" forces, and to find the acceleration due to free fall. He also tested the ratio of the gravitational and inertial masses of different materials, using a pendulum. Combining his second law (F = ma) with Hooke's law (F = -kx) it follows that the period of a pendulum for small swings is given by T = $2\pi\sqrt{l/g}$, just as Huygens found earlier using geometric reasoning and the idea of "centrifugal" force.

What experiments did Newton perform that suggested and confirmed his three laws of motion? Most physics teachers do not know the answer to this question and textbooks seldom discuss the experimental work of Newton beyond his optical experiments. It is not generally known that in his study of dynamics Newton used pendula to test his second and third laws of motion, as well as centripetal acceleration. Inertia, or his first law of motion, was seen as the consequence of a thought experiment that could not be experimentally tested directly. Newton, however, went beyond Galileo's idea of inertia as "the circumnavigation of an object on a perfectly smooth Earth" to the idea of "straight line motion with a constant speed in deep space when there are no forces acting on the object". His second law, F = ma, can be applied to a pendulum to demonstrate that if Hooke's law holds (restoring force is proportional to the displacement of the mass of the pendulum from the vertical) then we have simple harmonic motion. This part of the story is often told in textbooks, but Newton's experiments to test his third law is seldom mentioned.

© 2007, Arthur Stinner

The third law, “action is equal to reaction”, was demonstrated by Newton using two long (10–16 feet) pendula and having them collide. He used a result of Galileo (that the speed of a pendulum at its lowest point is proportional to the chord of its arc) and applied it to the collision by comparing the quantities mass times arc length, before and after collision. This is one of the few detailed accounts found in the *Principia* that high school students can read and understand. Students soon see that the third law is really equivalent to the principle of the conservation of linear momentum. Newton, of course, did not explicitly use the idea of the conservation energy or linear momentum in his work. Finally, Newton also used pendula to test the equivalence of inertial and gravitational mass and came to the conclusion that to a “thousandth part of the whole” they were equivalent. It is possible to replicate the experiments of Newton, using long pendula consisting of large wooden spheres. These experiments will be discussed in more detail later.

After Newton

The pendulum also played an important role in the next two centuries. Benjamin Robins (1742) adapted the pendulum in his ballistic device to measure the muzzle velocity of bullets.

IL 22 *** Complete description of the ballistic pendulum – including video

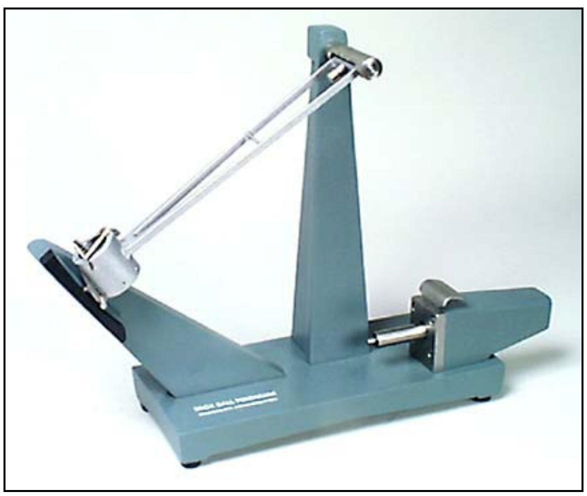


Fig. 9: A Modern Ballistic Pendulum

Count Rumford, famous as the debunker of the caloric theory, in 1781 adapted Robins’ method and patented it. This method of finding the muzzle velocity of bullets was used until the recent effective application of high speed photography. Here we have an experiment that can be replicated using a “Gauss gun” that propels ball bearings at low speeds. (To be discussed later).

IL 23 ** A detailed mathematical analysis of the ballistic pendulum

IL 24 ** Short biography of Benjamin Robins

IL 25 *** A detailed discussion of the ballistic pendulum—also features a calculator

© 2007, Arthur Stinner

Later (1790), the English physicist George Atwood, used the pendulum and incorporated it in his famous machine, named after him, as a research apparatus. One of the experiments he performed was to test Newton's second law of motion. Atwood's machine is forever enshrined in physics textbooks problems, but it is seldom mentioned that Atwood's approach was the first direct "test" of Newton's second law of motion. The pendulum in this experiment is part of the apparatus. A simple pulley can be used with two dissimilar weights and a pendulum to calculate the value of acceleration due to gravity.

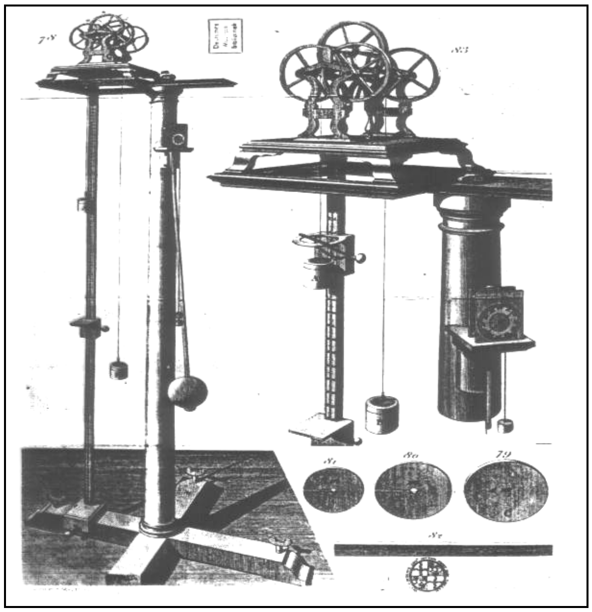


Fig. 10: Atwood's Machine

In 1851, the French Physicist Jean Foucault designed a very long and heavy pendulum to demonstrate for the first time directly that the Earth revolves around its axis. We can offer a good discussion of this dramatic and celebrated demonstration. Replication in the classroom is difficult but many science museums and centers have a Foucault pendulum demonstration.

© 2007, Arthur Stinner

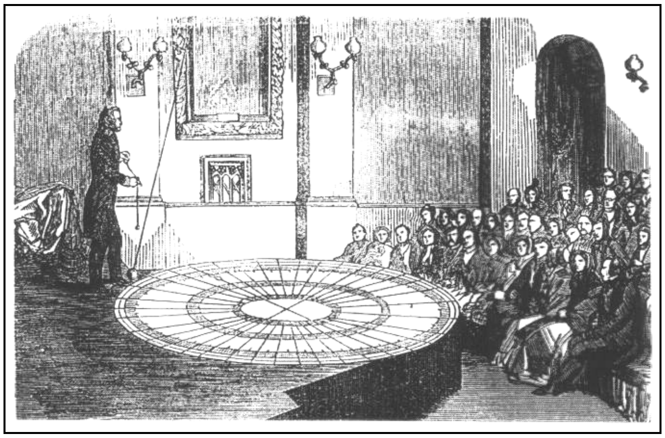


Fig. 11: Foucault Demonstrating his Pendulum in Paris, 1851

IL 26 **** Description of the Foucault pendulum

IL 27 *** Links to several Foucault pendulum descriptions (a variety of choices)

IL 28 ** Brief description of the Foucault pendulum

IL 29 *** Foucault pendulum applet showing forces present [French only]

IL 30 **** An applet of the Foucault Pendulum discussing Coriolis acceleration (advanced)

THE PRESENTATION OF THE CONTEXTS

The activity for the discussion of each of the six scientists should be introduced by the presentation of the short dramatizations found at the end of LCP 2, in the Appendices. You can also click on the "Conversations" for each scientist, as shown in the previous section.

Conversations with Great Scientists about Motion—A *Dramatization in Six Short Scenes.*

The presentation could be done by the students (with or without the assistance of the instructor). This presentation is designed to set the scene for the activities suggested for each scientist presented. The instructor could begin by having students (with or without the instructor) present. After the presentation and the discussion that accompanies such a group activity the class could go ahead with the activities below.

Aristotle and Motion

It is suggested that students begin with the presentation of the **Conversation with Aristotle**.

One often hears scientists or textbooks refer to the personal understanding of motion by students as "Aristotelian", suggesting that these views are simplistic and "unscientific". We must remember, however, that although his ideas about motion seem "unscientific" to us, Aristotle's

© 2007, Arthur Stinner

ideas about motion were a part of a complex philosophical system. Even today, these ideas seem very sound to the intuitive thinking of most people.

Background to Class Activities and Further Discussions

IL 31 *** A very comprehensive discussion of Aristotle's thinking in general

IL 32 ** A summary of Aristotle's ideas about motion on Earth

IL 33 *** A good summary of Aristotle's physics

Aristotle tried to find scientific principles and laws by keen observation and clear thinking but he admitted that he often relied on intuition (an educated imagination and intelligent guess). The idea of systematic experimentation will only be introduced around the beginning of the 17th century with the work of Francis Bacon and Galileo. But we will find that keen observation and rational thought can go a long way in understanding motion. Aristotle's ideas about motion were regarded as the final word and we will see that every philosopher and scientist up until Newton, including Galileo, had to come to terms with Aristotle' scientific thinking, in physics as well as in biology.

Aristotle's Scientific Thinking

Aristotle based his scientific reasoning on the following assumptions. He thought that these were self-evident and could not be questioned.

Aristotle' Foundational Assumptions about Physics:

1. **The world is rational and knowable.**
2. **Phenomena can be investigated by way of a deductive mode of reasoning.**
3. **There are *first principles* of physics that can be discovered, by observations and intuition.**
4. **We can only study terrestrial phenomena with success. Celestial phenomena will remain a mystery.**
5. **There are necessary, universal scientific facts that can be deduced from knowable principles.**

The following are Aristotle's *First Principles of Physics:*

© 2007, Arthur Stinner

Aristotle's First Principles of Physics

1. All motion is either natural or violent.

2. All natural motion is toward a natural place.

3. Violent motion is caused by the continuing action of an agent.

4. A vacuum is impossible.

The following are what Aristotle called the *Universal Propositions for Physics:*

Aristotle's Universal Propositions for Physics:

1 Light travels in a straight line.

2. All heavy objects fall toward the centre of the earth.

3. All opaque objects cast a shadow.

Predicates Proper to Physics:

4. Position, speed, resistance.

It is interesting that after Galileo these became measurable quantities and were used to establish laws of motion, expressed in mathematical terms. However, as we will see, this kind of physics would have been alien to Aristotle.

The following are what Aristotle thought were fundamental questions in physics:

Aristotle's Fundamental Questions of Physics:

1. What are the universal propositions and definitions that are proper to physics?

2. What kind of mathematical reasoning can we trust when describing the world?

3. Why do things move?

The following were the problems of physics for Aristotle:

© 2007, Arthur Stinner

Physics Problems for Aristotle:

1. How can I describe the motion of a freely falling object?

2. How can I describe the motion of projectile, a javelin?

3. What is a proper scientific argument?

Questions Based on Aristotle's Physics of Motion

Comment on Aristotle's scientific thinking by answering the following questions:

1. What do you thing Aristotle meant by "rational" thinking in describing the world? Perhaps today we would call it "objective", or "scientific" thinking. Discuss and comment.
2. Aristotle thought that celestial motion (essentially the motion of the planets and the stars) moved in response to laws that astronomers could never understand. However, Greek thinkers and astronomers, such as Pythagoras, Eudoxus before Aristotle, and Hipparchus, Aristarchus and Ptolemy after him, proposed very successful models to explain planetary motion. Aristarchus even suggested a sun- centered model that was revisited by Copernicus sixteen hundred years later. Look up the models proposed by these Greek natural philosophers and then set up discussion groups, and have each group report to the whole class.

IL 34 ** Visual representation of planetary motion and good applets

IL 35 *** Basic models of planetary motion

IL 36 *** Best source for the question above

3. Aristotle argued that the *First Principles of Physics* could be discovered by observation and intuition. We would call this kind of reasoning today "inductive" and "intuitive". Then, to explain everyday phenomena such as free fall, we would have to argue deductively from these principles to what he called a "scientific fact".

© 2007, Arthur Stinner

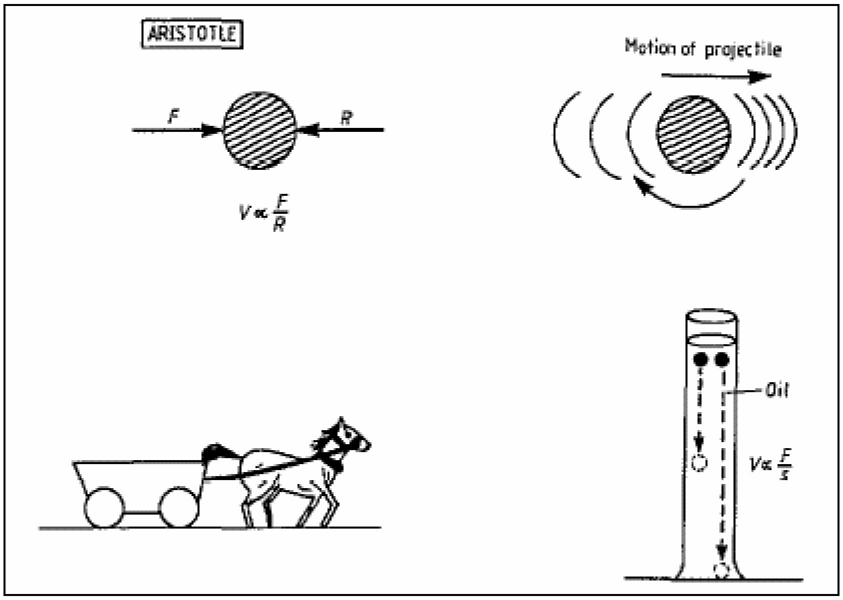


Fig. 12: Aristotle's Physics

<u>Questions and Activities for the Students</u>

1. What is an inductive argument and what is a deductive argument? Give an example for each in:
   a. Everyday discourse
   b. Mathematics/ geometry
   c. Logic
   d. Physics
2. What is considered an evidential argument in the following areas?
   a. Everyday discourse
   b. Mathematics/ geometry
   c. Logic
   d. Physics

IL 37 *** Discussion of inductive/deductive argument

IL 38 *** Discussion of inductive/deductive argument

The following are typical arguments that Aristotle considered scientifically valid:

© 2007, Arthur Stinner

Argument 1:

All heavy objects fall toward the center of the Earth.

This is a heavy object.

Therefore it will fall down vertically.

Argument 2:

Light in the heavens comes from the sun.

The moon and the earth are sometimes in line.

Light travels in a straight line.

All opaque objects cast a shadow.

Therefore: the round shadow on the Moon is the shadow of the Earth.

Another way to present this argument is as follows:

Why is there a round shadow on the Moon?

A round shadow is produced when a round opaque body (the Earth) is in the path of the sun's light that travels to the moon in straight lines.

"Oh, I see. The sun's light traveling toward the moon is intercepted by the earth and that is why there is a round shadow on the moon".

Discuss each of these. Do you find these arguments persuasive?

3. Discuss critically each of the four *First Principles of Physics* that Aristotle proposed. Use your own experience and knowledge of physics. Do you agree with Aristotle, or disagree with him?

4. Aristotle said that "it is not enough just to name a scientific fact. You must also know why it is a scientific fact". That is, we must back up a scientific claim with good evidence. Consider the following examples of "scientific facts":
   a. The Earth moves around the sun.
   b. The round shadow on the moon during a lunar eclipse is produced by the earth being in line with the sun.
   c. There is helium in the sun.
   d. The smallest part of an element is an atom.
   e. The moon has no atmosphere.

© 2007, Arthur Stinner

f. The distance to the nearest star is about 4 light years.

g. Your own examples.

For each try to give an evidential argument for believing these count as "scientific facts". Of course, Aristotle could have only answered the first two.

5. How, do you think, Aristotle would have explained the motion of an oscillating pendulum? Remember, he classified motion into two categories: *natural motion* and *violent motion.*

Questions for the Modern Physics Student

1. You are throwing a small metallic sphere straight up in to the air. What force acts on the ball:
   a. just after being released,
   b. at the midpoint going up,
   c. at the highest point of the trajectory, and
   d. on the way back toward the ground?

   How would Aristotle explain this motion?

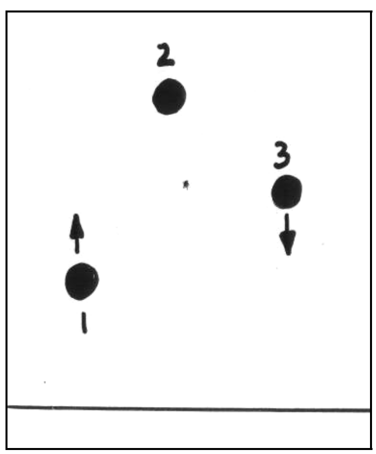


Fig. 13: Three Positions for a Ball Thrown Vertically. I

2. A rocket is in deep space, leaving the solar system. We know that to accomplish that, the rocket must leave the earth with a velocity of at least 33 km/s. What force must be acting on the rocket to maintain that velocity? How would Aristotle respond to this question?

© 2007, Arthur Stinner

The Pendulum in the Middle Ages

Oresme and the study of motion

It is suggested that students begin with the presentation of the **Conversation with Oresme**.

IL 39 *** Discussion of ideas about motion

IL 40 *** Excellent discussion of Oresme's ideas about motion

IL 41 *** A very good short biography of Oresme

It seems that the first detailed mention of the motion of a pendulum was made by Nicole Oresme, a noted 14th century natural philosopher. Oresme learned from his teacher Buridan that it is *impetus* that explains the motion of a projectile and a pendulum. Buridan was the first to decisively move away from Aristotle's ideas about motion. Oresme proposed a thought experiment:

> *A body falling through a well that has been drilled from one side of the earth, through the center, to the other end, would oscillate like a pendulum and eventually come to a stop at the center.*

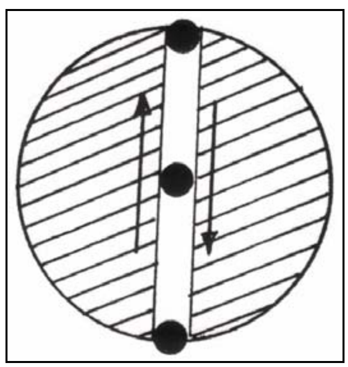


Fig. 14: A Modern View of Oresme's "Thought Experiment"

Class Activities and Further Discussions

John Buridan, the teacher of Nicole Oresme, developed the impetus theory further. He thought that an impressed force on a projectile was permanent unless acted on by resistances or other forces. He defined this impressed force as being proportional to the quantity of matter and the speed.

We must be careful, however, not to equate impetus with our concept of momentum. It is not clear, for example, whether he understood impetus as an effect of motion (as we might consider momentum), or as a cause of motion, which would make it similar to force. Buridan did not, however, arrive at a clear statement of inertia or of the conservation of momentum principle.

However, a pre-Galilean version of inertia was achieved by Oresme. Essentially, he argued that it is not possible to detect uniform rectilinear motion. He also thought that the earth

© 2007, Arthur Stinner

rotated and agreed that the air and water share the motion. The conservation of linear momentum principle, however, was not clearly stated until Descartes.

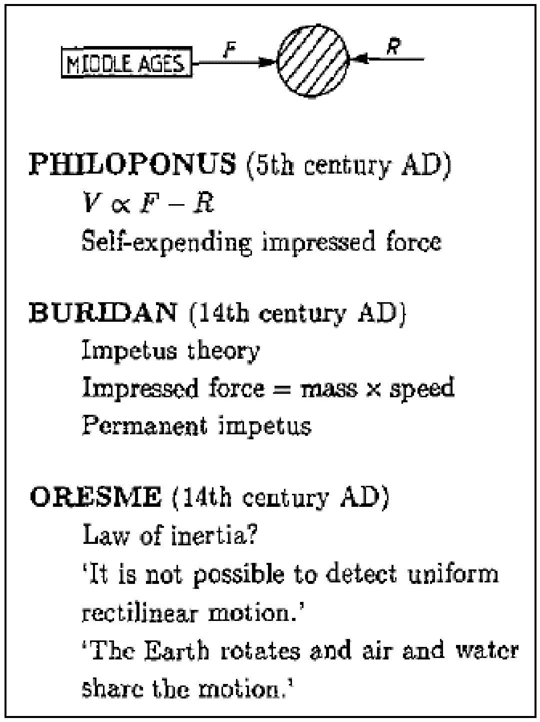


Fig. 15: Middle Ages: Explanation of Motion

Questions for Discussion

1. Discuss Buridan's impetus theory. The pressing question was: "What happens to the *impressed force*" on an object that is being thrown?" Another way to put this question would be: "What happens to the force that accelerated the object to velocity at the moment the object leaves the hand or the device that launched it?

   Discuss this question in small groups and then report to the class.

2. It may be instructive to consider the question posed earlier with Aristotle and see this motion through the ideas of Oresme. It is well known that most people (including many physics students!) believe in a personal impetus theory, much along the lines of Oresme. You are throwing a 100 g metallic ball straight up in to the air. What force acts on the ball,

   a. just after being released,

   b. at the midpoint going up,

   c. at the highest point of the trajectory, and

   d. on the way back toward the ground?

© 2007, Arthur Stinner

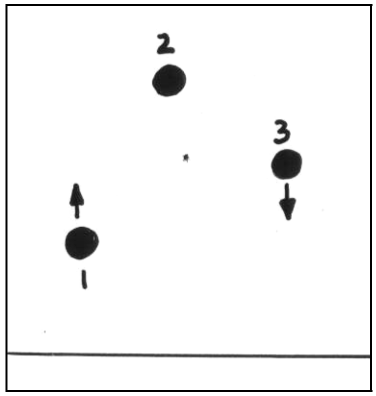


Fig. 16: A Ball Thrown Vertically, Seen at Three Positions II

<u>Questions and Problems for the Modern Physics Student</u>

1. Why do you think Oresme's ideas about motion seem more acceptable to us today than Aristotle's?

<u>"Thought Experiments" for the Student</u>

("Thought experiments" will be defined and discussed a little later)

1. Imagine a tunnel through the earth, as shown in Fig. 16. If you dropped a heavy object from the surface of the earth down this tunnel, describe the motion. Neglect frictional effects.
2. Now imagine a pendulum, with the length of the radius of the earth. Compare the motion of the pendulum to that of an object falling through the earth.

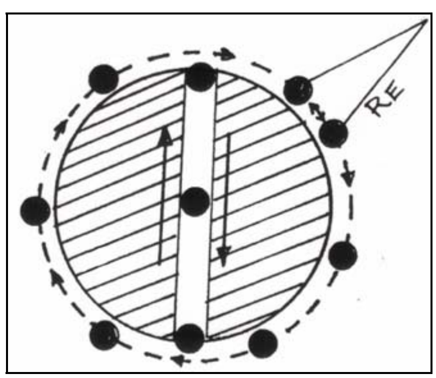


Fig. 17: Motion in a Tunnel through the Earth

Note: This problem will be considered again, after discussing Newton's physics.

© 2007, Arthur Stinner

Galileo Describes the Motion of a Pendulum

IL 42 *** A well written short history of Galileo and the pendulum

IL 43 *** A video showing a simple pendulum experiment

It is suggested that students begin with the presentation of the **Conversation with Galileo**.

Galileo was the first to show that trajectory motion is parabolic and that the period of the pendulum is fairly constant. He proved, and later confirmed by experiment that the period is proportional to square root of the length. He also argued that the arc of a circle represented the least time (brachistochrone) path of an object descending along a frictionless path in a vertical plan. He then used his discoveries about the pendulum to understand motion better.

Background to Class Activities and Further Discussions

Galileo changed the approach to studying motion and moved from a qualitative to a quantitative description. He studied Aristotle and was acquainted with the ideas of Buridan and Oresme. In fact, one of the results of medieval physics of motion was the mean speed theorem that Galileo used as the starting point for his mathematical description of free fall.

Galileo's Scientific Thinking

Following Aristotle, Galileo declared that the world is rational and could be understood. But he went further. He believed that:

> *The world must be studied not through secondary qualities but through primary qualities imbedded in the language of mathematics.*

Examples of secondary qualities are colour, shape, smell and the feeling of hot and cold. Primary qualities are length, speed, acceleration and temperature. Galileo argued that secondary qualities are not measurable but the primary ones are.

Galileo went beyond Aristotle's approach of finding universal principles of logic and physics and arguing deductively. He was looking for mathematical (geometric) description of phenomena (such as free fall and the period of a pendulum) that could be confirmed by systematic experimentation.

Like Aristotle, Galileo made assumptions about the world. His foundational assumptions were:

Galileo's Foundational Assumptions about Physics:

1. **The world can be understood by rational thought.**
2. **The language of Nature is the language of mathematics (geometry).**
3. **The world must be studied only through primary (measurable) qualities of length, time, area, etc.**

© 2007, Arthur Stinner

Discussion

1. Galileo followed Aristotle's "rational" method in trying to understand the phenomena of the world. But he went beyond Aristotle. For him "rational" meant that phenomena should be investigated using logic, geometry and experiment. His approach added the necessary condition of careful experimentation to confirm or reject hypotheses. Aristotle's physics was based on "naked eye" observations only.

2. Galileo also went beyond Aristotle by insisting on using only "primary ", that is measurable quantities such as distance, speed, acceleration, time, area, volume, temperature.

   These assumptions were connected to measurable primary qualities, which lead to the following questions:

Galileo's Questions about Physics of Motion:

1. What is the mathematical law that correctly describes free fall?

2. How can we decompose complex motion, like the motion of a projectile, to simpler motions?

3. Can we describe such complex motion as the motion of a body through a resistive medium?

4. How can we describe the oscillation of a pendulum?

5. What are the primary qualities that allow us to describe motion quantitatively?

Discussion

1. Contrary to what many textbooks claim, Galileo did not investigate accelerated motion experiment without having a good idea (hypothesis) about the kind of motion he expected to find.

2. Fig. 15 reflects the general opinion before Galileo which followed largely Aristotelian lines of reasoning but incorporating a later idea of "impetus" by Oresme. He argued that an object shot from a cannon, for example, followed a straight line until it "lost its impetus," at which point it fell abruptly to the ground.

Galileo's insight was that the motion of a projectile can be separated into two independent motions that are perpendicular to each other, one vertical, and the other horizontal (see LCP1). The vertical motion is simply a free fall motion and the horizontal motion is a constant velocity motion. Of course, the frictional effects of the air are neglected here.

IL 44 *** Trajectory motion as seen by impetus theorists and as understood by Galileo.

© 2007, Arthur Stinner

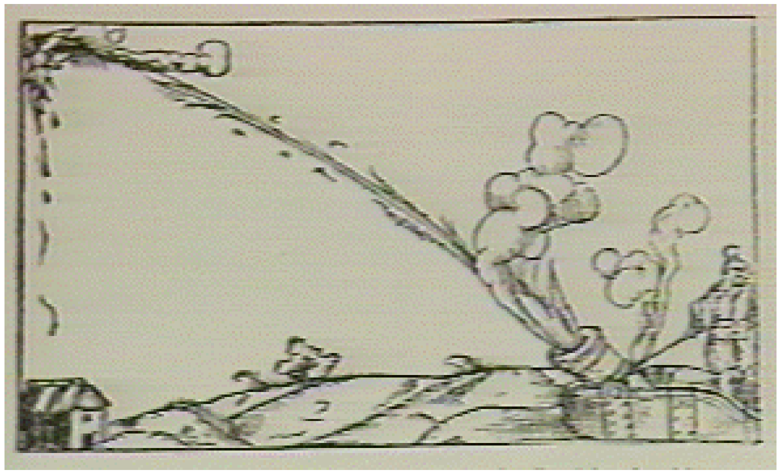


Fig. 19: Pre-Galilean Thinking About Projectile Motion

3. Galileo tried to understand motion through a medium (such as a sphere falling in water) but was unable to describe this motion. It was Newton who was first able to describe this complex motion. We will see in LCP 3 that motion the force on a car is proportional to the square of the velocity.

Galileo and Thought Experiments (TEs)

We have already encountered a TE when we discussed the Oresme's imaginary object falling through the Earth.

TEs have a long history and a natural connection to the development of physical concepts. They can be traced back to Zeno's paradoxes of motion and Aristotle's program of explaining phenomena guided only by naked eye observation and rational thought. We will se shortly how Galileo championed TEs to show that motion, as observed on a ship moving with a constant speed in a straight line, cannot be distinguished from motion observed at rest. He also presented TEs to argue that the motion of heavy objects in free fall are identical and constructed TEs for his students to clarify the concepts of speed and acceleration.

We will see later that Newton described two TEs in his *Principia* in order to demonstrate his notion of absolute space. In the late 19th century, Austrian physicist Ernst Mach developed the idea of the "Gedankenexperiment" (German for TE) to include them in physics education, and Einstein raised TEs to a high level of abstraction to clarify his ideas in both theories of relativity and in probing the foundations of quantum theory.

TEs can also take on different roles in our understanding and development of physics. Some TEs shake the very foundations of a theory and promote debate, often over extended periods of time. Examples of this kind of TEs are Galileo's argument to refute Aristotle's claim that the rate of descent of an object in free fall is independent of weight, Newton's bucket experiment to argue for absolute space, and Einstein's elevator in space to show that gravitation and acceleration are equivalent. Other TEs can be used to clarify familiar concepts or situations or to extend the concepts to new and diverse situations. TEs of this type often respond to the

© 2007, Arthur Stinner

question: “what if…?” These “what if” type of TEs will be both interesting and informative for your students to debate. “What if the Olympics were held on the moon?”, “What if an object fell through a “tunnel” that goes through the centre of the earth?” are all examples of such TEs.

<u>The following are the two most famous TEs by Galileo:</u>

1. Galileo used a TE to show that all heavy objects, regardless of their weight, fall at same rate. He argued that according Aristotle, heavier bodies fall faster than light ones. But what would happen if a if a heavy cannon ball were attached to a light musket ball? He now shows that reasoning in the manner of Aristotle would lead to an absurd conclusion. First, the light ball will slow down the heavy ball (acting as a kind of drag), so the speed of the combined system would be slower than the speed of the heavy ball falling alone. But the combined system is heavier than the heavy ball alone, so it should fall faster. But that is absurd, for how can the heavy ball be both faster and slower than even the combined system? Thus, Aristotle’ theory of free fall is wrong.

2. Galileo used a TE to argue that you cannot tell the difference between constant speed and rest. Galileo argued that if a sailor dropped a canon ball from the mast of a ship that was smoothly moving with a constant velocity, the cannon ball would fall directly below him. Whatever the sailor would do, assuming a smooth constant velocity in a straight line, he would not be able to tell that the ship was in motion if he confined his attention to the ship. Using modern terminology, Galileo thus claimed that *there is no distinction between constant motion and rest.* All of us have experienced this when traveling in train on a level track moving with a constant speed.

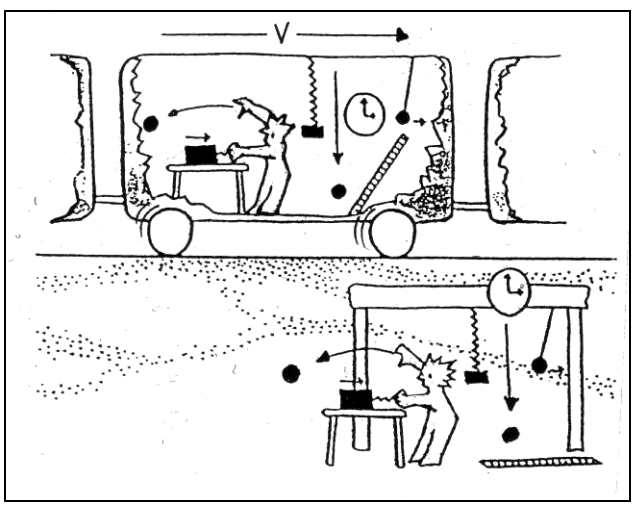


Fig. 20: A ‘Thought Experiment’: Rest and Constant Motion Are Equivalent

IL 45 *** An excellent applet to illustrate Galilean inertia.

© 2007, Arthur Stinner

3. To illustrate the idea of inertia, Galileo first imagined a ball rolling down an inclined plane and then rolling up another one. See IL . He argued that when the ball finally rolls on a level surface, in the ideal case when there is no resistance, it would roll around the earth and continue to do so forever. See Fig. 21.

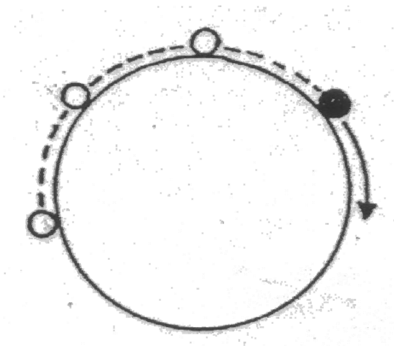


Fig. 21: Galilean Inertia

IL 46 *** An advanced discussion of inertia and motion, with history.

Questions Galileo asked:

1. How does the speed of a freely falling object vary? Can I express this as a proportionality statement?
2. How does the period of a pendulum vary with the length?
3. How can I describe the motion of a horizontally projected object?

How did Galileo answer these questions? How did he argue and what experiment did he perform? Answer these questions the way Galileo would have answered them.

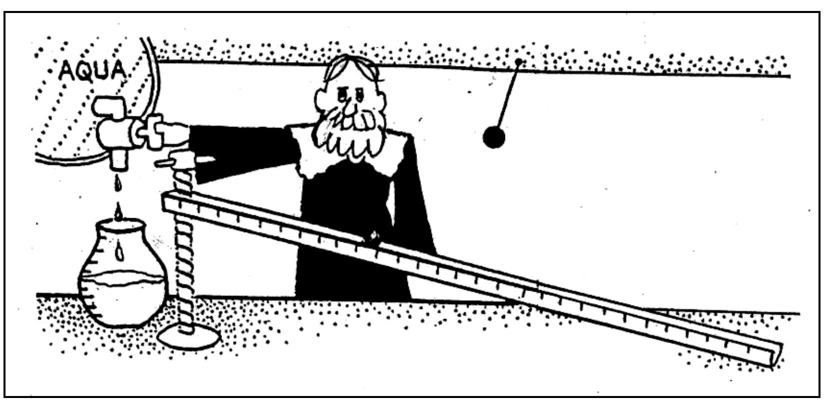


Fig. 22: Galileo Determines the Acceleration of Free Fall

Research Problems for Galileo

© 2007, Arthur Stinner

1. Galileo proved that the time of descent of an object (on a frictionless surface) along an incline represented by any chord (see Fig. 18) is the same and is equal to the time it would take for an object to fall through a distance of twice the radius.

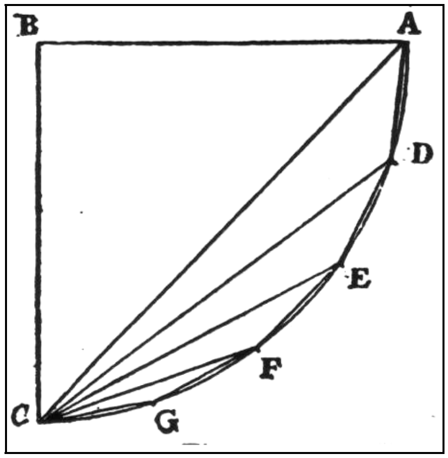


Fig. 23: Galileo's Geometric Argument

2. Galileo showed that the period is proportional to the square root of the length and believed that the arc represented the least time (brachistochrone) of an object descending along a frictionless path in a vertical plane.

   Using a sketch, show Galileo's argument.

<u>Questions for the Student</u>

The following questions are based on on the excellent internet link, IL 44. Study and listen to the Video in which a replication of Galileo's famous inclined plane experiment can be seen.

IL 47 ** Survey of the inclined plane

IL 48 **** An excellent video replicating Galileo's inclined plane experiment

1. How did Galileo slow down motion so he could measure the motion of a falling object? What was this argument to show that the motion along any inclined plane is always a motion described by constant acceleration?
2. How is the time of descent measured in the video?
3. How did Galileo actually measure the time intervals?
4. Make a chart that shows the pattern Galileo discovered when he measured the units of distance a falling object covers in each unit of time.
5. Use a sketch to explain what "this beautiful numerical progression—one, three, five, seven" means. What is special about the sums of odd integers?

© 2007, Arthur Stinner

6. Summarize in your own words how Galileo arrived at the final mathematical statement that the distance a heavy object falls is proportional to the square of the time of fall.

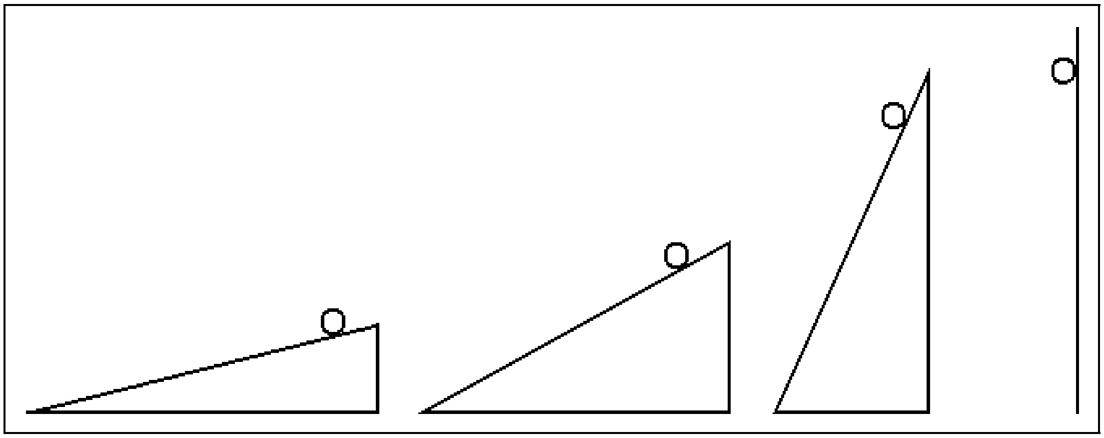


Fig. 24: Descent on an Inclined Plane and Free Fall

IL 49 *** IAs to study the inclined plane: an interactive applet.

Problems for the Student, Based On **IL 49**

The interactive applet (IA) allows you to change the angle and the coefficient of friction of the inclined plane. Get acquainted with this excellent IA by looking at various motions. . Study the motion of the object, first with the coefficient of friction equal to zero (totally smooth motion, just as Galileo imagined it) and then by using various values for the coefficient friction, up to 0.30. Notice that for each run you can find the actual distance along the incline in meters.

1. Galileo's inclined plane had an angle of about 8°. Find the acceleration of an object for this setting. Show that it would be about 1.4 m/s^2. Check this value from the IA.
2. Show that the acceleration for 30º would be ½ of g, or about 5 m/s^2. Check this value with the IA.
3. Find the angle for which an object would just begin to slide for a coefficient of friction of 0.3. Confirm your result with what the IA predicts.
4. In an actual experiment, using a wooden block, the coefficient of friction is found to be 0.10. Calculate the acceleration along a 30º plane and check the answer with the prediction made by the IA.
5. Make up your own problems and check them, using the IA.

IL 50 *** Galileo's arguments, taken from his own writings.

IL 51 *** A very simple set up to check the period of a pendulum.

© 2007, Arthur Stinner

IL 52 *** Same as above.

Problems for the Student (Based On IL 52)

1. First, study the IA, finding out how you are able to change the various parameters (length, gravity , angle and mass).
2. What should be the length of a pendulum to produce a period of 1.0 second on earth?

On the Moon? On Mars? Check your answer.

Research Experiments for the Student

1. Test the periods of a pendulum (about 1 m long) for small angles using different masses. Does the amplitude make a difference? Does the mass?
2. Compare the period of a pendulum of two lengths, one twice the other. Before making an observation predict what the ratio of the times will be. (Make sure you use the correct definition of a *period.*)
3. In 1639, the mathematician Mersenne, who also studied the theory of the pendulum, argued against Galileo that the period of the pendulum was not isochronous:

   And if he (Galileo) had merely counted the thirty or forty oscillations of the one pulled aside twenty degrees or less and the other eighty or ninety degrees, he would have known that the one having the shorter swings make one oscillation more in thirty or forty oscillations.

   Test this statement by Mersenne using a pendulum of about 1m length and determine the period for a small angle (about 5 degrees) and a large angle (about 45 degrees) and comment.

A Special Research Experiment for Galileo

It is not well known, that Galileo used the pendulum to measure free fall directly**.** Galileo knew that the period of the pendulum was nearly constant (for small angles of displacement) and used this to measure the actual time it took for a metallic sphere to fall from a height of about 3 meters (of course, he used *cubits*, not meters). He found "the ratio of the distance fallen from rest to the length of the pendulum that was timing that fall by swinging the vertical through a small arc". (See Fig. 25.)

> *A pendulum was held out from the vertical board and then released simultaneously with the dropping of a small metallic sphere. Galileo adjusted the height from which the metallic sphere fell until the "thud" of the sphere hitting the floor coincided with that of the pendulum hitting the wall. Galileo found that the ratio was 1.108.*

Research Experiment for the Student

Try to replicate this experiment with another student. (This experiment should probably be done for the whole class by the instructor, assisted by students). You can easily replicate this clever and simple experiment by Galileo. Use a length of the pendulum of between 2 and 3

© 2007, Arthur Stinner

meters. Find the ratio of the height from which the sphere fell to the length of the pendulum. You must, of course, remember that Galileo did not have a watch to time the period of the pendulum. How did he find the period of the pendulum?

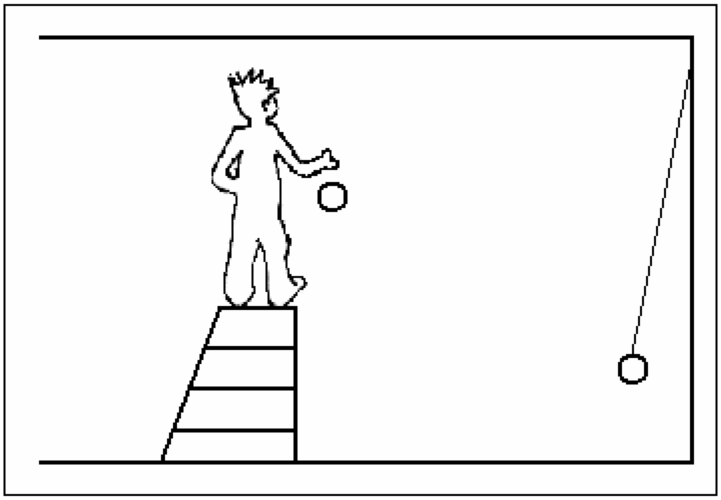


Fig. 25: Using ¼ of a Pendulum's Period to Measure Free Fall Time

<u>Problems for the Student</u>

1. Using the modern formula of the period of a pendulum ($T = 2\pi\sqrt{l/g}$), show that the ratio that Galileo should have found is about 1.23. What was the % error of your result?
2. Galileo concluded that the ratio he found was a constant for all heights of fall, at all places on earth. We might add that it would be the same on the moon.

   Why must this be true?
3. Galileo was interested in finding the distance a freely falling (heavy) object travels in one second. Calculate that distance.
4. However, we are more interested in finding the value of g (the acceleration due to gravity, in m/s^2) using Galileo's method. Calculate this value. How close is your value to the accepted value of about 9.8 m/s^2.

Huygens Uses the Pendulum as a Research Instrument

IL 53 *** Information on motion of the pendulum and the motion along curves, like the cycloid

IL 54 *** Experimental setup to show different paths yield different times through the same change in height

IL 55 *** A very good discussion of the motion along a cycloid

It is suggested that students begin with the presentation of the **Conversation with Huygens**.

© 2007, Arthur Stinner

The Dutch natural philosopher Huygens was the first to find the modern formula, namely that $T = 2\pi\sqrt{l/g}$ for the simple pendulum and also the first to write the mathematical statement for "centrifugal" acceleration as $a = v^2 / R$, around 1670. He used long and heavy pendula to determine the value of gravitational acceleration. He later correlated latitude and the local value of g to test his ideas. Huygens was also the first show (geometrically) that the path along which a pendulum would show isochronous motion was a cycloid. Later (about 1700), Johannes Bernoulli, Newton and others showed that not only is this path isochronous but it is also the least time descent of an object in a vertical plane between two points.

IL 56 ** A brief biography of Huygens

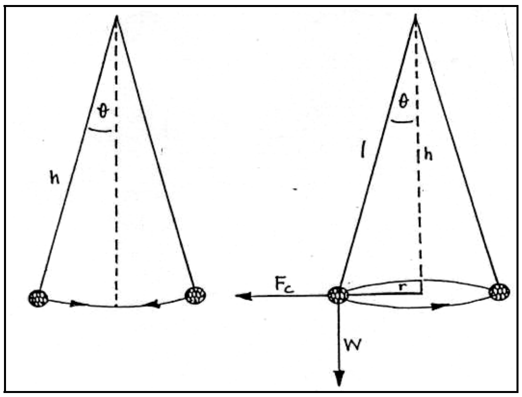


Fig. 26: The Conical Pendulum

IL 57 **** Applet of the conical pendulum

IL 58 *** Applet of a conical pendulum

Huygens and His Experiments

Huygens was the first to show (geometrically) hat the path along which a pendulum will produce isochronous motion is a cycloid. Look up the definition of a cycloid and shape a wire that follows the cycloid and place it under a wire that is straight (see IL 54). Also shape a wire that is an arc (1/4 circle). Have a heavy bead or nut slip down the wires at the same time convince yourself that the cycloid is the "least time" (Brachistochrone) path.

Research Problems for Huygens

1. Knowing that the period of a conical pendulum (for small displacements from the vertical) is the same as that of a pendulum with length equal to the height, he showed that the "centrifugal" acceleration of the pendulum is given by $\mathbf{v^2 / R}$. Here v is the linear velocity, or $2\pi R / T$, T the period and R the distance from the vertical to the pendulum sphere. See Fig. 26.

© 2007, Arthur Stinner

Research Experiments for Huygens

1. Huygens rotated a conical pendulum of length l (when the angle θ was small) he found that the time for one rotation was about equal to the period of a pendulum with length L. Test this in the lab. See Fig. 26.

   Equating the "centrifugal" force with the component of the weight acting along the radius of the circle he was able to find the modern formula for the period of the pendulum: T = $2\pi\sqrt{l/g}$. Try to show this.

2. Using this formula Huygens tested the value of g for various latitudes.

   a. Find the expression for the value of g in terms of the other variables.

   b. He wanted to find the effect of the rotation of the earth. What would the difference between g as measured on the North Pole and at the equator be, assuming a perfectly spherical earth with a constant density?

   Research Experiment for the Student

   1. Using the formula found by Huygens find the local value of the acceleration due to gravity (g) by determining the period of oscillation of a long (about 2 meters) pendulum attached to a very heavy mass (at least 1 kg). How many significant figures are you allowed to express your answer to?

   Problems for the Student

   1. Find the length of the pendulum that has a period of 1 second.
   2. Show that a 1 meter pendulum has a period of close to exactly 2 seconds.
   3. On a certain planet the period of a 1 m pendulum 1.50 seconds. What is the gravitational attraction on the surface measured in m/s2?
   4. Calculate the length of a chandelier in a cathedral that swings with a period of 3.0 seconds.

IL 57 ** The pendulum as a time keeping device.

Questions for the Student

1. Since a 1 m pendulum has a period of very close to 2 seconds, why, do you think, was this fact not used in the definition of the meter? Is it only a coincidence?
2. Calculate the time keeping accuracy of a pendulum clock, assuming that you have a three–figure accuracy in the period.
3. You have landed on a planet and you want to find out what the local gravity is measured in m/s^2. You make a pendulum using a metallic sphere and a string of 1 m length and find that the time for 1 period is about 2.9 seconds.

   a. What is value of g for this planet?

   b. You then test this value by dropping the sphere through a distance of 10 m.

© 2007, Arthur Stinner

How long would it take for the sphere to fall through this distance?

c. If your mass is 70 kg what would be your weight on this planet?

Robert Hooke Discovers the Force Law That Produces Simple Harmonic Motion

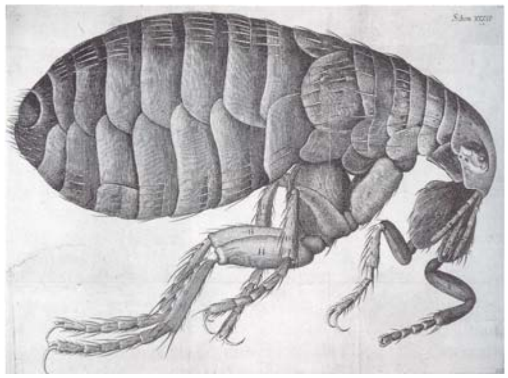


Fig. 27: Image Hooke Drew Using His Microscope

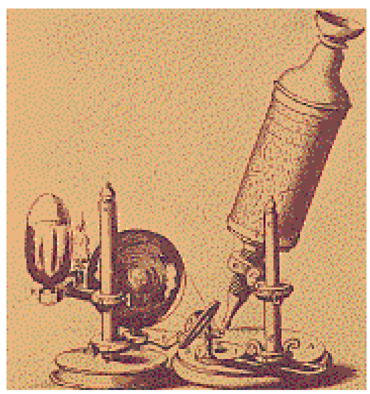


Fig. 28: Hooke's Microscope

IL 58 *** Source of Fig. 22

IL 59 * Source of Fig. 23

IL 60 *** Brief biography of Hooke

IL 61 *** A very comprehensive discussion of Hooke: biography and his work)

Robert Hooke used his law (F = -kx) to show that simple harmonic motion (SHM), like that of the pendulum, or an oscillating mass attached to a spring, arises when this law holds. His scientific battles with Newton were legendary. When Newton became the president of the Royal Society in 1705, he expunged all vestiges of Hooke. Thus we have no likeness of him. We identify Robert Hooke by the famous drawing he made in his revolutionary "Micrographia" that he published at the age of 30. See Figs. 27 and 28.

© 2007, Arthur Stinner

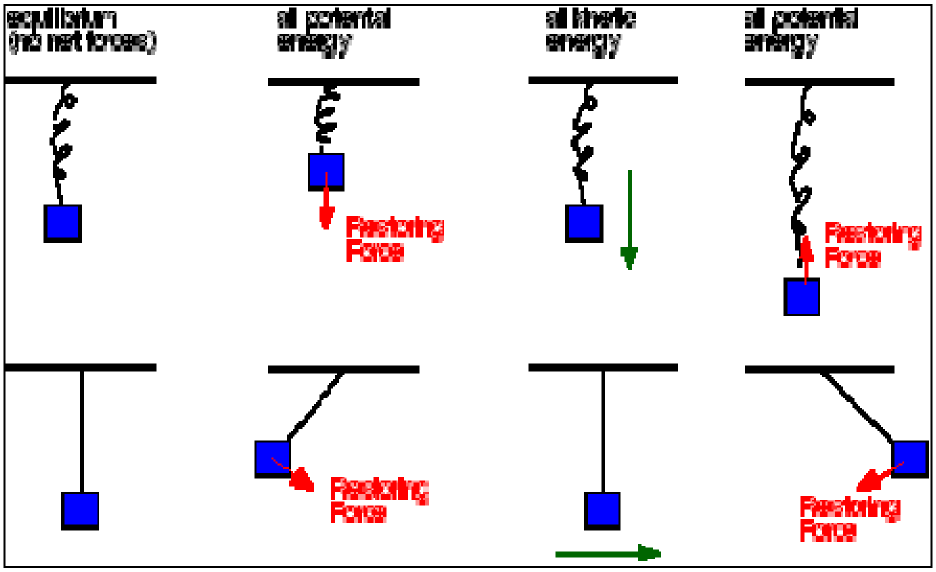


Fig. 29: A Spring and a Pendulum Oscillating In Unison

An Experiment Hooke Performed

1. Hooke had to show that when a spring was extended the length of extension was directly proportional to the force applied. He soon discovered that this law only holds within the limit of what we now call the "elastic limit".

Research Experiment for Students

1. Attach a spring scale to a heavy pendulum and pull the pendulum to about 10° and note the reading. Then continue to pull the pendulum through larger angles. Does Hooke's law apply to large angles? (Note: when Hooke's law does not apply we do not have SHM).

Research Questions for the Student

1. How would you demonstrate that the oscillating motion of a mass attached to a spring is the same motion as that of a pendulum?
2. SHM is all around us. Water being displaced in a U-shaped tube, and a small boat being displaced vertically by waves in water are examples. Can you think of others? See Fig. 29.

An Experiment for the Student

1. Use a spring and various weights to confirm Hooke's law. Use a spring with a mass attached to it, so that it oscillates about once a second. Now find the length of a pendulum that swings with the same period.

© 2007, Arthur Stinner

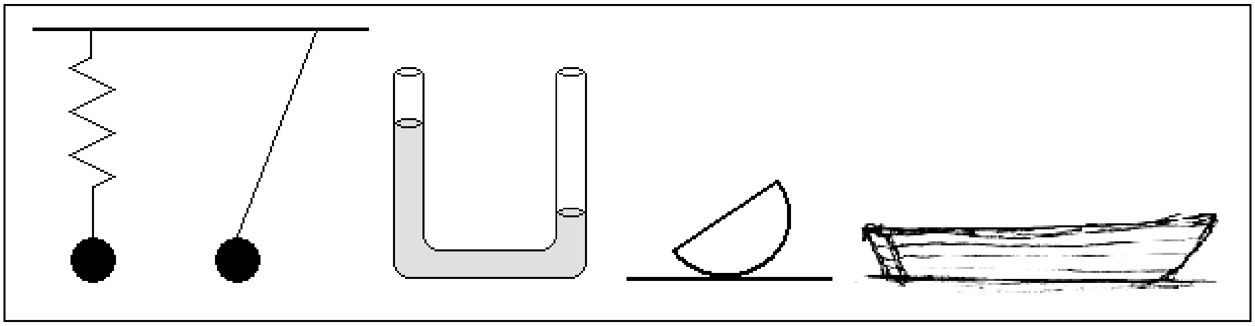


Fig. 30: Examples Of SHM: Spring, Pendulum, Liquid In A U-Tube, Wedge Block, Rocking Boat

IL 62 *** Hooke's law applet

IL 63 *** Illustrations of Hooke's law

IL 64 *** Advanced discussion of SHM

IL 65 *** IA for the pendulum and SHM

IL 66 *** IA for the pendulum and SHM

Newton Uses Pendula to Test His Laws of Motion

IL 67 ** Short biography of Newton

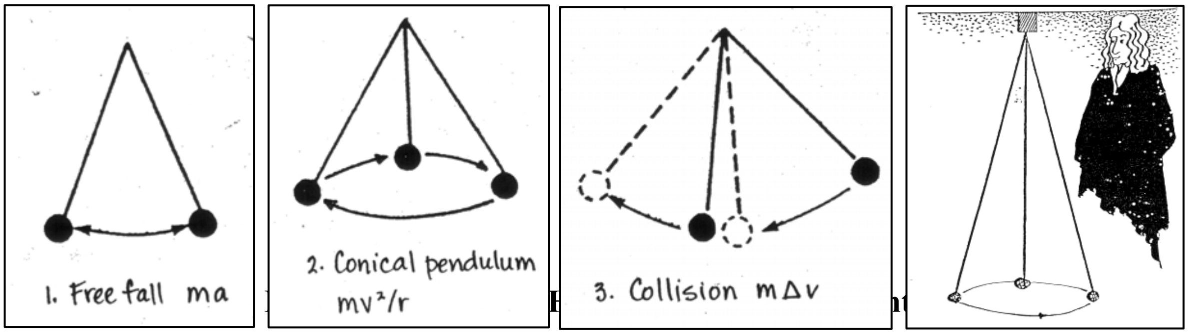
IL 68 ** Short biography of Newton

IL 69 *** A scholarly and detailed biography of Newton.

It is suggested that students begin with the presentation of the **Conversation with Newton**.

In his study of dynamics (motion that considers the effect of forces) Newton used pendula to test his second and third laws of motion, as well as centripetal acceleration. Inertia, or his first law of motion, was seen as the consequence of a thought experiment that could not be tested directly. Newton went beyond Galileo's idea of inertia as "the circumnavigation of an object on a perfectly smooth Earth" to the idea of "straight line motion with a constant speed in deep space when there are no forces acting on the object". His second law, F = ma, can be applied to a pendulum to demonstrate that if Hooke's law holds (restoring force is proportional to the displacement of the mass of the pendulum from the vertical) then we have simple harmonic motion. The third law, "action is equal to reaction", was demonstrates by him using two long (10–16 feet) pendula and having them collide. He used a result of Galileo (that the speed of a pendulum at its lowest point is proportional to the chord of its arc) and applied it to the collision by comparing the quantities mass times arc length, before and after collision. The third law then must be seen as containing the principle of the conservation of linear momentum. See Fig. 31.

© 2007, Arthur Stinner



Discussion of Fig. 31

Using a pendulum, Newton had available three distinct sets of observations that could be connected to three distinct meaning of the notion of force. The ordinary oscillating pendulum in part 1 is simply constrained free fall and the force here is given by the formula F = ma; the conical pendulum in part 2 is described by the centripetal force $F = mv^2 / r$, and in part 3 we see colliding pendula (they collide head-on) a situation described by the third law of action = reaction, where action and reaction is the same as the momentum. The force that each sphere experiences is given by equating impulse I ($I = F\Delta t$) with $m\Delta v$.

In the three tables below you will find a summary of the physics of dynamics (mechanics) of Newton, in terms of Assumptions, Questions, Problems, and New Questions that Newton presented in the *Principia.*

Table I

Newton's Foundational Assumptions for physics:

1. Mathematics is the core of description and explanation in physics

2. Mass points interact by way of a central force.

3 Space is Euclidian.

4 Time is absolute.

5. Mass points interact instantaneously.

Discussion

1. Like Galileo, Newton believed that the language of science (physics) is mathematics. We must, however, be careful here and remember that when Galileo says "mathematics" he means geometry and Euclidean ratios. As we have seen, in his arguments about motion he uses geometry and ratios only, and not algebraic equations, as we now do. Newton in his *Principia* also uses complicated geometry most of time, but occasionally does present algebraic expressions and equations. Today we use predominantly algebraic expressions to describe motion.

© 2007, Arthur Stinner

2. The gravitational force between masses, say the moon and the earth, act as if the masses were concentrated at their centers.

3. Space can be described by Euclidean geometry. That means that if you measured the angles in a triangle that connected three points in space, however far apart would always contain three angles that added up to 360º. Of course, if you measured the angles of a triangle connecting Toronto, New York, and Chicago, the sum of the angles would be more than 360º. (We will discuss this later in more detail).

4. Newton believed that:

Absolute, true, and mathematical ***time****, from its own nature, passes equably without relation the anything external, and thus without reference to any change or way of measuring of time (e.g., the hour, day, month, or year).* (Taken from the *Principia*).

That means that if a measure a time interval in my laboratory, in a satellite orbiting the earth we would measure exactly the same time interval. Einstein showed in 1905 that time is not absolute. We will discuss the arguments of Einstein in LCP 10.

5. According to Newton, a planet orbiting the sun would feel the effect of the gravitational pull instantaneously, that is, *as if* the gravitational force traveled with an infinite speed. Newton did not like this assumption but was forced to make it. He called it an "occult" phenomenon. Newton said:

That one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one another, is to me so great an absurdity that, I believe, no man who has in philosophic matters a competent faculty of thinking could ever fall into it.[4]

We will discuss this phenomenon in LCP 10.

Table II

Foundational Questions for Newton:

1. **What are the fundamental physical quantities in terms of which we can describe the dynamics of free fall, collision and centripetal force?**
2. **How should we define accelerating force? As m Δv,** or **$F = mv^2/r$**, or **F = ma?**
3. **How can we describe the gravitational force and the motion of a planet around the sun?**
4. **Do the laws of motion on earth (terrestrial laws) also govern the motion in of the planets (celestial motion)?**

© 2007, Arthur Stinner

Discussion

1. The fundamental quantities Newton are an extension of those Galileo used to describe kinematics, that is, the study of motion without considering the forces involved. Galileo introduced the definition of speed, uniform acceleration and distance, showed how these were related in free fall motion.

The new quantities that Newton needed to introduce were the ideas of force momentum and impulse.

2. Newton struggled with these different ways of expressing the idea of force.

3. One of Newton's great achievements was that, using his three laws of motion and the inverse square law of gravitational attraction, he was able to calculate the motion of the planets. We will discuss this a little later.
4. Aristotle believed that the laws of motion on earth (terrestrial physics) and the laws of motion guiding the planets (celestial physics) are different. Newton, however, showed that the same laws apply to the earth as well as to celestial motion.

The central problems he solved were:

Table III:

Newton's Foundational Physics Problems:

1. **To show that Galileo's law of free fall is just a special case generated by the second law of motion.**
2. **Show that the motion of a pendulum (for small angles) can be described by using Hooke's law and the second law of motion.**
3. **To find the mathematical expression for force in collisions, straight line motion, and circular motion.**
4. **To find the nature of the path of a planet obeying the inverse square force law.**
5. **To show that Kepler's laws are derivable from the laws of motion and the universal gravitational law.**
6. **To show that a spherically homogeneously distributed mass has the same gravitational effect as a point equivalent mass.**
7. **To show that the gravitational mass of an object is the same as its inertial mass.**

© 2007, Arthur Stinner

Discussion

1. Examining free fall, using Newton's second expressed as F = ma, we find that the motion of the freely falling heavy object must be such that a = g. Show this.

2. Use a force diagram to show that the restoring force (produced by gravity) acting on an oscillating pendulum can be considered linear, up to about 8–10 degrees.

3. Newton's argument here can be understood if we considerer circular motion. See IL 71a. A very comprehensive discussion is given in I.B. Cohen's article (see references).

4. Newton showed in the *Principa* that an inverse square law for gravitational attraction and using the sun as the source of a central force will necessarily produce an ellipse. You can consult text books that deal with classical mechanics. Also see IL 71b.

5. See I.B. Cohen (references).

6. Newton solved this problem using his discovery of calculus (what he called "fluxions"). The main idea here is that, if we consider the earth as a perfect sphere, with constant density, it can be shown that the gravitational effect on an object outside the earth (or on the surface of the earth) would be the same 'as if all the mass were concentrated at the center'. This effect is called Newton's shell theorem. See IL 70.

7. We showed (see Discussions 1.) that in free fall a heavy object falls with a constant acceleration of a = g. Show that this can only be true if the gravitational mass m_g is equal to the inertial mass m_i .

IL 70 ** An advanced discussion of Newton's 'shell theorem'.

IL 71 ** A brief history of Newton's motivation to write the *Principia.*

IL 71a *** Newton's Laws and Kepler's Laws.

IL 71b *** Newton applies his laws to planetary motion.

Table IV:

Foundational Experiments for Newtonian Dynamics:
1. Investigating the dynamics of the simple pendulum, using Hooke's law.
2. Investigating collision (head-on) with pendula.
3. Investigating the behavior of a conical pendulum.
4. Using Atwood's machine to test the second law (F = ma) (~ 1780)
5. Using a Cavendish set up to find the gravitational constant G of gravity (~1790).

Textbooks seldom discuss the experiments that Newton performed to test his laws of motion. He used pendula for each of the experiments mentioned below. It is interesting to note

© 2007, Arthur Stinner

that Atwood's Machine was used about 100 years after the publication of the *Principia* in 1684 to directly experimentally confirm Newton's second law of motion. The inverse square law of gravity had its first celestial confirmation at about the same time made by Cavendish, at about 1780/90.

Table V:

Thought experiments Newton proposed and discussed:
1. First law of motion thought experiment
2. The Rotating Bucket thought experiment.
3. The Rotating Globes in a Void thought experiment.

The "thought experiments" mentioned below are discussed here. The student should realize that it is not really possible to experimentally confirm the first law. Why not? We have already discussed the other two thought experiments.

Table VI:

Newton's Questions for the Future:
1. Why are inertial and gravitational masses equivalent?
2. Can force and mass be expressed in a non-circular way?
3. How can we quantitatively demonstrate the way particles of matter in motion endowed with forces produce the observed phenomena in nature, for both small and large-scale phenomena?

<u>Discussion</u>

1. The fact that heavy objects fall with the same acceleration in the earth's gravity means suggests that inertial mass of an object (m_i) and its gravitational mass (m_i) are equivalent.

   Newton tested this using a pendulum with different masses and different materials (wooden and various metallic spheres) and found that the period of the pendulum for a given setting does not change. In LCP 10 we will discuss how this equivalence became the foundation principle for Einstein's General Theory of Gravity.

2. Newton understood that, using his second law, F = ma (or, as he expressed it $F = \Delta (mv) / \Delta t$) can be used to express mass as m = F / a. How can you escape this circular definition, connecting mass and force? He did not know the answer.

3. This question was asked by Newton toward the end of his life. He was only acquainted with gravitational force. Static electric and magnetic forces were known, but not understood. In modern physics we have four forces that have been identified.

© 2007, Arthur Stinner

What are these forces? Look up the answer to this question and discuss it with your instructor and other students.

Questions for the student, in reference to the above.

The following questions should be discussed in small groups, and then later a report should be given to the whole class.

1. Newton showed that the second law of motion, conventionally written as F = ma, can be applied to all motion. Apply this law to an object in free fall and to a pendulum. Discuss.

2. Study Fig. 31 that shows how the pendulum can be used to illustrate all three aspects of the notion of force, F = ma (the accelerating force on an oscillating pendulum), F = v^2 / R (the centripetal force on a conical pendulum), and $F \Delta t = m \Delta v$, (the impulse on a pendulum during collision). Consider the following problems:
   a. A 1 m long pendulum with a ball of 1 kg attached to it is pulled 5 º to the vertical.
      i. Find the force and the acceleration of the ball at the beginning of the motion.
      ii. Show that the accelerating force (for small angles) obeys Hooke's law.
      iii. Show that the acceleration of the ball must vary sinusoidally.
      iv. Calculate the period of the pendulum using Huygens' formula above.
   b. The 1 m pendulum is now used as a conical pendulum, that is, the ball is revolving in a circle with an angle of 5º to the vertical.
      i. What is the centripetal force acting on the ball?
      ii. What is the tension in the string?

Thought Experiments of Newton

Following Galileo, Newton presented a thought experiments that is very famous and should be discussed. We can, for now, think of thought experiments as "a devices of the imagination used to investigate the nature of things". We will look at Newton's famous "rotating bucket" thought experiments below.

Newton spent a great deal of effort trying to explain this privileged status of inertial frames and postulated the existence of absolute space and time. He thought that inertial forces provide a clear indication of absolute motion. To illustrate how absolute motion could be determined he presented two thought experiments, his famous bucket experiment and the rotation of two globes in an immense void experiment. He introduced the bucket experiment as follows:

If a bucket is hanging from a very long cord and is continually turned around until the cord becomes twisted tight, and if the bucket is thereupon filled with water and is at rest along with the water and then, by some sudden force, is made

© 2007, Arthur Stinner

to turn around in the opposite direction and, as the cord unwinds, perseveres for a while in this motion; (Newton [1726] 1999, 412-413).

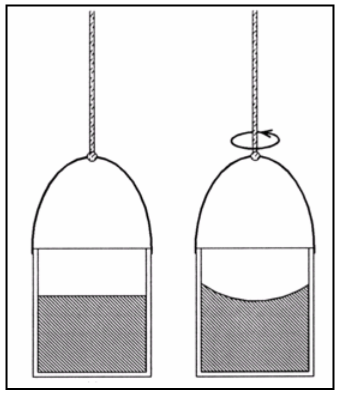


Fig. 32: Newton's Bucket

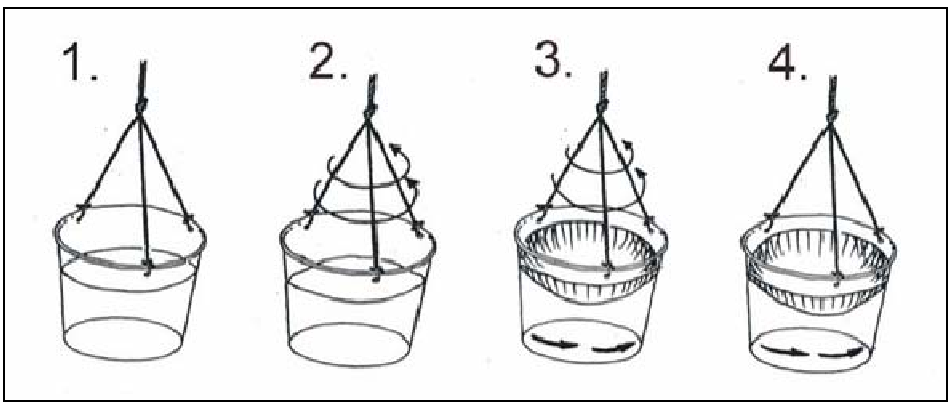


Fig. 33: Behavior of Water in the Newton's Bucket

IL 72 *** Source of Fig. 32.

IL 73 **** An excellent source for thought experiments – source of Fig. 30

IL 74 *** Advanced discussion of the thought experiment

IL 75 ** Advanced discussion of Newton's thought experiments and Mach's principle

Newton continues to describe four stages of the bucket experiment. Initially, the bucket is filled with water, the cord has not been released, and the surface of the water is level (Fig. 33.1). In the second stage the cord begins to unwind, there is a relative motion between the water and the bucket, and the water is observed to be level (Fig. 33.2). Next, the water catches up with the sides of the bucket and there is no relative motion between the water and the bucket, and the water is observed to be concave (Fig. 33.3). Finally, the bucket stops, the water is spinning relative to the bucket and the water is observed to be concave (Fig. 33.4). At stage 2 and at stage 4, the bucket and the water are in motion relative to each other. However, in the first case the water has a level surface and in the second case the water has a concave surface. It appeared that

© 2007, Arthur Stinner

the shape of the surface of the water was not dependent on the relative motion of the water and the sides of the bucket. Newton concluded that it was the spin of the water with respect to absolute space that mattered. If the water was not spinning with respect to absolute space then its surface was level, but when it was spinning with respect to absolute space, the surface of the water was concave.

Newton Experimented with Pendula

1. Newton showed that the periodic swing of a pendulum, obeying Hooke's law illustrated the second law F = ma, where F = -kx. Free fall itself was regarded as a special case of the second law, where inertial and gravitational masses are considered equivalent. Discuss with your fellow students and your instructor.

2. Newton tested the equivalence of inertial and gravitational mass by using pendula of the same weight but made from different materials. Using the equation of the period of the pendulum for small angles, show how this can be done.

3. Newton also tested his third law; commonly written as "for every action there is equal and opposite reaction". An excerpt from the *Principia* is given below, as well as the picture of the experiment that he performed. Discuss Newton's experiment and his mathematical and geometric argument. (See Fig. 34).

IL 76 * Source of Fig. 34

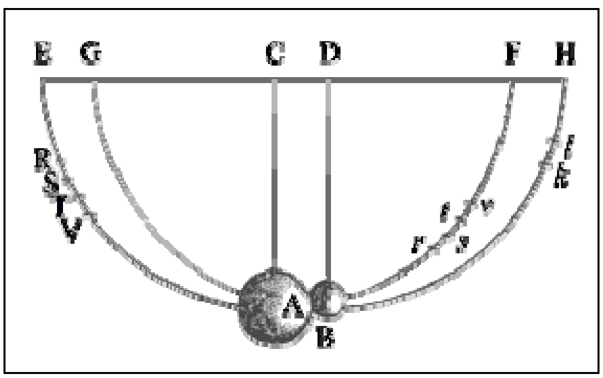


Fig. 34: Newton's Sketch of Colliding Pendula from the *Principia*

4. What forces act on a pendulum, at the moment it is released, at the midpoint, at the bottom? This question was posed earlier, but know we should be able to answer it guided by an understanding of Newton's laws of motion. Note: this is a difficult question and the students should discuss it among themselves as well as with the instructor.

IL 77 * Source of Fig. 32

© 2007, Arthur Stinner

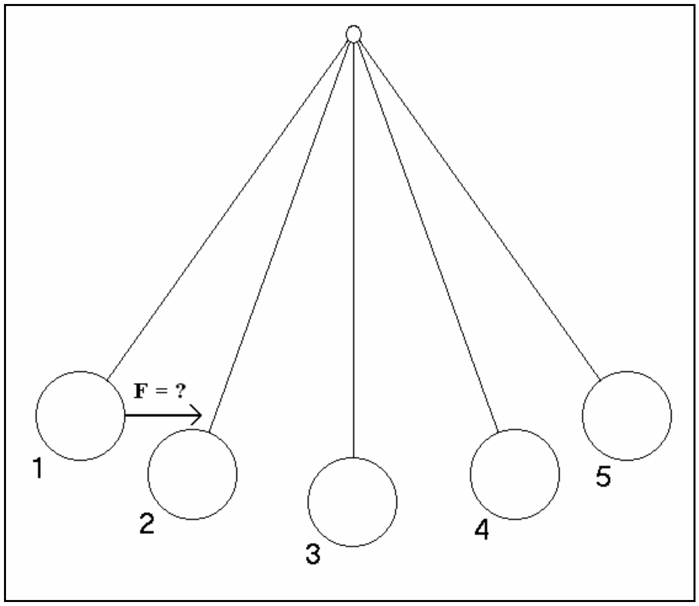


Fig. 35: Force Acting on a Pendulum II

(Note: This is a difficult problem. Students should be able to answer this at the end of this LCP).

Newton's Third Law of Motion and the Conservation of Linear Momentum Principle

Newton did not use the idea of conservation of linear momentum, or the idea of kinetic energy, nor the idea of gravitational potential energy in the *Principia*, as we do today. His third law as demonstrated by the collision of two wooden spheres, as he analyzed this, amounted to the same thing. Today we would say: In an elastic collision the mementa before impact are equal to the momenta after impact. Remember, momentum is a vector quantity and kinetic energy is a scalar quantity.

The following problem will illustrate how these quantities, momentum (P = mv) and kinetic energy (1/2 mv^2), and potential are connected. It should be added, at this point, that Newton did not write down his second law as F = ma, as we do today. He expressed his second law in terms of a change of momentum, F = m Δ v / Δ t. This way of expressing the second law shows the intrinsic connection between the seconds and the third law. It was the Swiss mathematician Leonhard Euler who first expressed the second law as F = ma in the 1740s. This way of writing the second law then became the conventional way of writing it.

Two wooden pendula, with masses of 1 kg and 2 kg collide, as shown below. Assume a perfectly (head-on) elastic collision. The 1kg ball is pulled up high enough so that it collides with the 2 kg ball with a velocity of 3m/s.

i. Find the initial velocity of the large ball after collision.

ii. Find the velocity of the small ball after collision

iii. What was the kinetic energy of the small ball before collision?

© 2007, Arthur Stinner

iv. What is the kinetic energy of the balls after collision?

v. If the string attached to the balls is 1 m long how high will the balls rise?

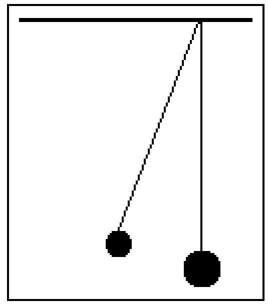


Fig. 36: Collision of a 1 kg Wooden Ball onto a 2 Kg Wooden Ball

Note: Collision experiments using long wooden pendula, are also described in the "Conversation with Newton".

<u>More Thought Experiments</u>

1. For Newton the first law of motion is connected with the idea of "straight line motion with a constant speed in deep space when there are no forces acting on the object".

   Try to imagine this situation as an extension of Galileo's thought experiment that illustrated inertia, who described this motion as motion without a resistance acting on an object.

8. We have already discussed Newton's thought experiment of the cannon on top of a high mountain in LCP 1. In the *Principia* Newton sketched a picture of the motion of an object that was launched by way of a super canon from a high mountain, about 20 miles high. He thus anticipated satellite motion around the earth, more than 300 years ago. See Fig. 33 below.

© 2007, Arthur Stinner

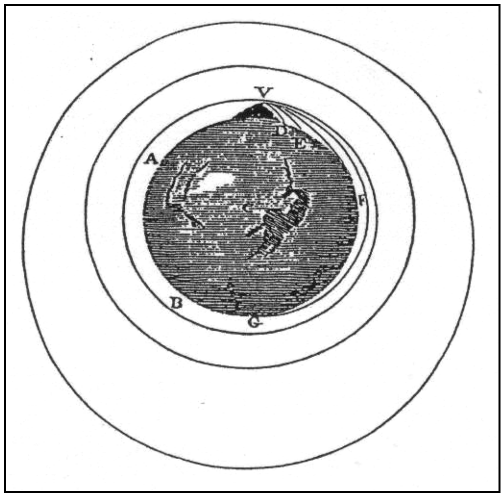


Fig. 37: Sketch of a High Mountain Launching of High Speed Projectiles

IL 78 *** The cart experiment (applet) to illustrate Newton's second law

IL 79 ** Newton's laws –a brief historical presentation

IL 80 *** Details about Newton's cannon thought experiment

IL 81 *** An excellent interactive program that Newton would have enjoyed.

IL 82 *** Article by a science historian

Questions Newton Could Have Answered About Satellite Motion

Use IL 81 for answering these. (Note: You can obtain a qualitative and some quantitative understanding of satellite motion using this interactive program. Below we will discuss a quantitative approach, using Newton's own calculations.)

1. Use the program in IL 81 to find the velocity required to place a satellite into a circular orbit, not far above the surface of the earth.

2. Notice that velocities lower and higher than this velocity describe motion that looks like an ellipse.

3. Find the escape velocity of an object from earth, the velocity necessary to escape the gravity of the earth.

Question for Space Physics

(In preparation for future LCPs, where we will discuss interplanetary travel).

1. A rocket with a mass 100 tons is in deep space, leaving the solar system. To accomplish that it had to be accelerated to a velocity of about 32 km/s relative to the sun. What must be the force acting on the rocket to maintain this velocity?

© 2007, Arthur Stinner

2. Most textbooks discuss the escape velocity from the earth, but few discuss the escape velocity from the solar system. Remember, the earth revolves around the sun, in a near circular orbit, at about 30 km/s. What is the escape velocity?

Newton Calculates the Period of the Moon

According to Newton himself, he calculated the period of the moon using his laws and the inverse square law of gravitational attraction in 1666, when he was only 23 years old. Historians, however, are now convinced that he actually calculated this much later, probably around 1681/2, when he started the writing of his *Principia.* See IL below.

Read what science historians J J O'Connor and E F Robertson (See IL 83) say in their article:

> *Fifty years after these events Newton was to record his own recollections of these events which, although interesting, do not really agree with the known historical facts! [I preserve Newton's old English.].* Newton wrote (in 1705):
>
> *In the same year I began to think of gravity extending to ye orb of the Moon and… from Kepler's rule of the periodical times of the Planets being in sesquialternate proportion to their distances from the centres of their Orbs, I deduced that the forces wch keep the Planets in their Orbs must reciprocally as the squares of their distances from the centres about wch they revolve: and thereby compared the force requisite to keep the Moon in her Orb with the force of gravity at the surface of the Earth, and found them answer pretty nearly. All this was in the two plague years of 1665-1666...*

The noted Newton scholar (see references) Bernard Cohen had strong evidence to believe that Newton wanted to "ensure his own priority in discovering the inverse law of gravitation" he stated in 1717 that he discovered this law as early as 1666, when in actual fact he discovered it during the time of the writing of his *Principia* in about 1684. Cohen believed that Newton could not have discovered it that early (he was 23 years old) because Newton did not have a clear understanding of centripetal force.

IL 83 *** Newton's theory of Universal Gravitation

A Historical Description of Newton's Calculations of the Period of the Moon

1. Rewrite the quotation by Newton in modern English. Discuss your version with other students and the instructor. Then continue with the following problem:

2. Try to verify Newton's claim that, using Kepler's third law (See IL 83) it is possible to show that …*the forces which keep the Planets in their Orbs must be reciprocally as the squares of their distances from the centres about which they revolve.*

   In other words, Newton claimed that combining the empirical (observational) fact of Kepler's third law of planetary motion with the definition of centripetal acceleration, he could show that an inverse square law describes the force of gravity between the sun and the planets.

© 2007, Arthur Stinner

(Kepler's third law: T^2 / R^3 = a constant, where T is the period of the planets and R the distance to the center of the earth and the expression for centripetal acceleration for circular motion is $a_c = v^2 / R$, where v is the velocity of the satellite and R the distance between the planet and the sun. Remember, Kepler's laws apply to the elliptical motion of planets, but it also applies to circular motion, because a circle is just a special case of an ellipse).

3. After finding his fourth law, (namely the inverse square law of gravity), Newton applied it to the motion of the moon around the earth.

   Try to follow the argument in IL to find out how Newton calculated the period of the moon. Remember that this is a modern version of how Newton made this calculation.

Note: We will use these ideas again in greater detail in LCP 8, where we will discuss the motion of Mars and how to get to there.

Problems for the Student

1. Show that the period of a satellite just above the earth's surface would be about 84 minutes.

2. Show that a pendulum would have a period of 84 minutes if its length were the radius of the earth. Why?

3. Now look at the motion of the pendulum through the eyes of Newton: Use his physics to explain this motion. Specifically, show the unbalanced force acting on the pendulum at A, B, C, D, and E. A is the starting point and C the lowest point. See Fig. 38 below.

4. Finally, compare the explanations for the motion of the pendulum, as seen by Aristotle, Oresme, Galileo, and Newton.

   (This is a good exercise to show that we see phenomena around us through our ideas and concepts.

© 2007, Arthur Stinner

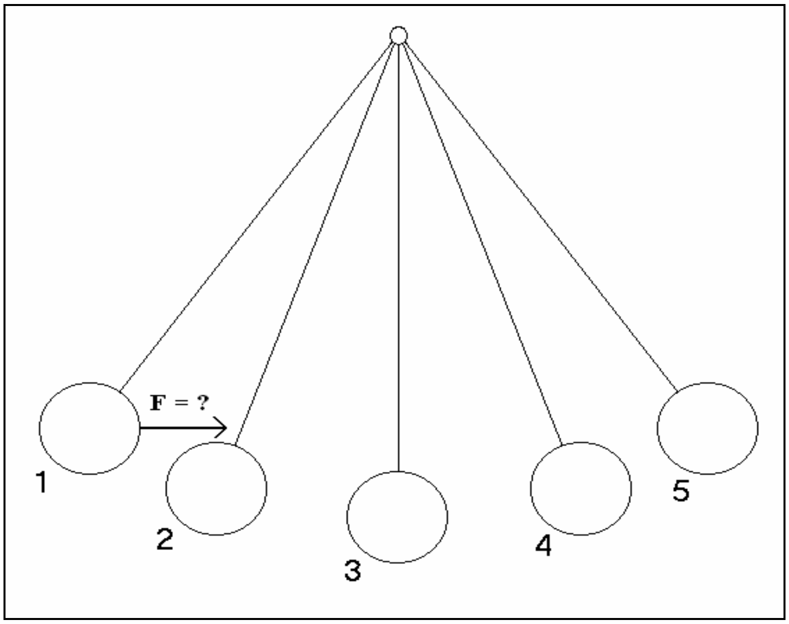


Fig. 38: Forces Acting On an Oscillating Pendulum. III.

IL 84 **** Applet on pendulum motion

George Atwood Invents His Machine to Be Used As a Research Instrument

IL 85 *** IA for Atwood's machine

George Atwood (1745-1807) was a celebrated lecturer at Trinity College, England. He was best known for his book, *A Treatise of Rectilinear Motion*, a contemporary textbook on Newtonian dynamics. He was the inventor of the famous Atwood's Machine, used until the beginning of the 20th century for experiments in Newtonian dynamics. It was used for both research and teaching. To find the time of descent of the larger mass he used a pendulum clock. See picture below.

© 2007, Arthur Stinner

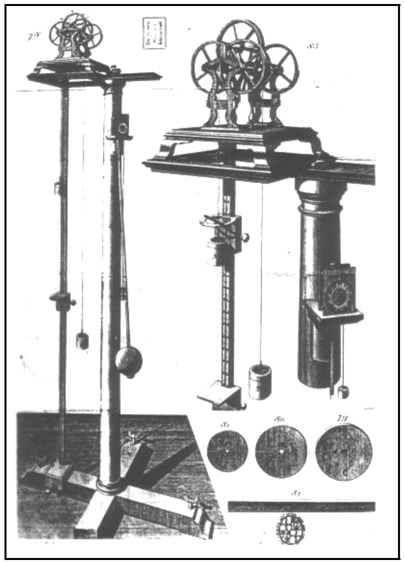


Fig. 39: Atwood's Machine Revisited

IL 86 ** Biography of Atwood

IL 87 *** Details and pictures of Atwood's machine

Research Experiment for the Student

Check Newton's second law of motion by using a long pendulum (about 2 m) and hang it against a wall so that you can determine the time for 1/4 period. (Exactly as you have done finding the acceleration due to gravity for Galileo's pendulum). Hang two masses from a pulley as shown below. For a given height (about 1 m) adjust the weight difference until the time of fall is equal to the time it takes the pendulum to hit the wall. Alternatively, you can fix the weight difference and adjust the height. This is a simplified version of an experiment that Atwood performed over 200 years ago. (See Fig. 40).

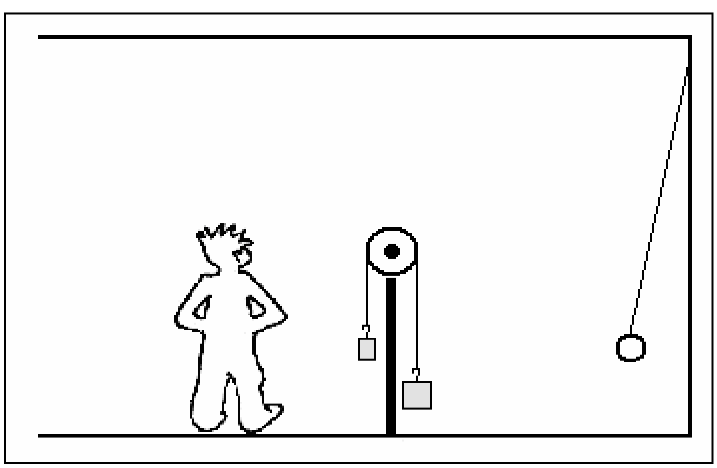


Fig. 40: Setup for Student Research Experiment (Above)

© 2007, Arthur Stinner

Problems for the Student

1. Using Newton's second law, show that when two masses m_1 and m_2 are suspended, where m_1 is larger than m_2, then the acceleration of the system is given by $a = g (m_1 - m_2) / (m_1 + m_2)$.

2. What must be the ratio between the masses so that the acceleration of 0.5 g? Test this prediction experimentally.

An Experiment for the Student

1. Use IL 85, which is an interactive program to study Atwood's machine, to find the acceleration for various masse attached to the strings. Compare some of these values with those you obtain using the formula above.

2. Using a simple pulley, set up an experiment, as shown In Fig. 40 above. Place two weights, one larger the other, and release them, at least 1 m above the floor. Predict the acceleration of the system and then test your prediction. Comment.

A Thought Experiment for the Student

Compare the result of the Atwood machine experiment on Earth with a similar experiment on the Moon. Would you find that the formula for the acceleration of the system $a = g (m_1 - m_2) / (m_1 + m_2)$ you found in problem 1 above would still work?

Discuss.

Leon Foucault Uses a Pendulum to Confirm the Rotation of the Earth

IL 88 *** Discussion of Foucault's pendulum

IL 89 **** An excellent link with a good video of the Foucault pendulum

IL 90 ** Foucault pendulum on the South Pole!

IL 91 *** Detailed description of Foucault's pendulum with images

It is suggested that students begin with the presentation of the **Conversation with Foucault.**

Newton, as well as, Galileo was aware that the rotation of the Earth should influence the motion of moving bodies. Newton even suggested an experiment to show that the Earth rotated on its axis. However, not until the middle of the 19th century was it possible to show directly that the Earth rotates. This was accomplished by the French the French physicist Leon Foucault (1819-1868)

Foucault was famous for showing that light traveled faster in air than in water. This was important because it convinced physicists that Newton's corpuscular model of light had to be replaced by the wave model. He invented the gyroscope and , built a number of telescopes and was the first to use photography to study the sun's surface. But his most famous accomplishment was the design of his pendulum to show that the Earth rotates around its axis.

© 2007, Arthur Stinner

According to William Tobin (Scientific American article) the inspiration for the experiment with a pendulum came when he happened to twang a steel rod that was clamped in the chuck of a lathe. He noticed that although the although the rod rotated with the chuck, the plane of vibration did not (see Fig.). This is a discrepant event to most can be explained by appealing to Newton's first law. Foucault reasoned chuck and the rod were analogous to the Earth and a pendulum.

He first experimented with a 5 kg brass pendulum, 2 meters long. He eventually built a pendulum that was 67 meters long with a 28 kg sphere. The pendulum was installed in the dome of the Pantheon in Paris and a public demonstration give. The pendulum slowly swung clockwise and had to be restarted every 6 hours. During this time the pendulum veered the expected 60 to 70 degrees clockwise. In 1851 he thus publically provided the first direct proof that the earth rotates. Foucault showed that it was the Earth that was revolving and not the pendulum that produced the apparent rotation! See Fig. 35 and 36.

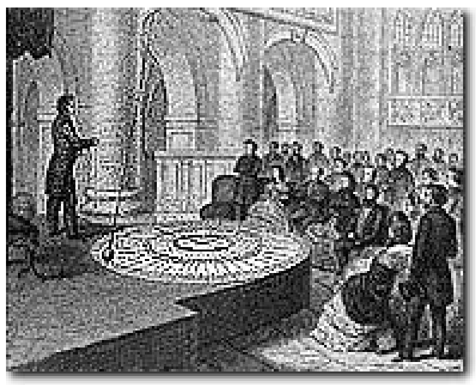


Fig. 41: Foucault Demonstrates His Pendulum In 1851.

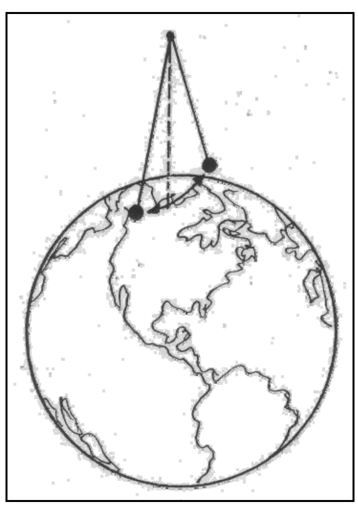


Fig. 42: Foucault's Pendulum on the North Pole

IL 92 ** Source of Fig. 41.

© 2007, Arthur Stinner

Class Activities and Discussions

1. A hundred and fifty tears before Foucault's famous demonstration of the rotation of the Earth, Newton pointed out that if the earth rotated then it should be possible to directly test this motion the following way: Drop a heavy object down a deep well and check the point of impact and see were it hits the bottom of the well. (See Fig. 40.) It is interesting that he failed to think of using a long pendulum to demonstrate the rotation of the Earth.

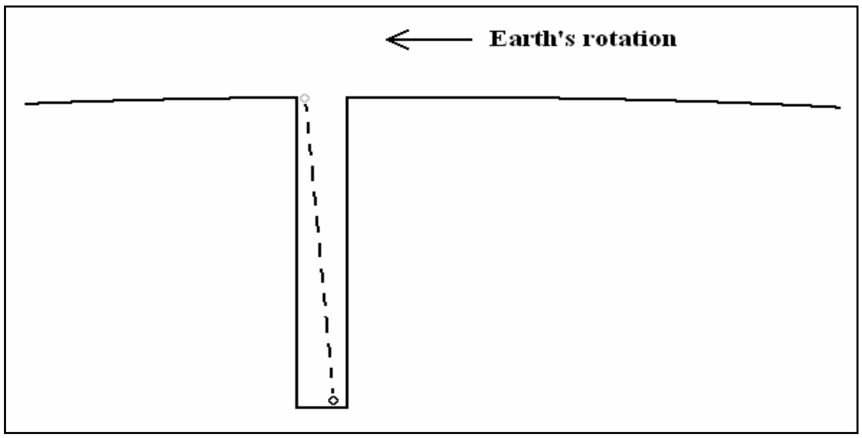


Fig. 43: A Sketch of a Falling Object in a Very Deep Well

IL 93 An applet for the FP

IL 94 A video of the FP

Problems for the Student

1. Where would a heavy object fall, relative to the vertical, if the well is 100 m deep? How could Newton have tested this?

2. What was the period of Foucault's pendulum (FP)?

3. First, give an argument to show that the period of rotation at any latitude θ, given by T = 24 h / sin θ is a plausible answer.

4. How long would it take for a FP to rotate 360°, on the North Pole? On the equator?

5. Calculate the period of rotation of a FP in Winnipeg, Toronto, on the equator, Sydney, or the city or town you live in.

Research Questions for the Student

1. We have talked about two methods (ways) to show that the Earth rotates. There are others. Describe some of these.

© 2007, Arthur Stinner

Research Experiment for the Student

1. Construct a Foucault pendulum, using a heavy bob and a long strong wire, and convince others that the Earth rotates.

Revisiting Oresme's "Tunnel-Through-The-Earth" Thought Experiment

One day Isaac Newton received a letter from Robert Hooke. In this letter, Hooke outlined the mathematics governing how objects might fall if dropped through hypothetical tunnels drilled through the Earth at varying angles. Though it seems that Hooke was mostly interested in the physics of the thought experiment, an improbable yet intriguing idea fell out of the data: a dizzyingly fast transportation system.

See the ILs below. The complete history and the solution to the problem can be found in these links.

IL 95 ***

IL 96 ***.

Thought Experiments for the Student

1. Imagine a tunnel through the earth, as shown below. Drop a heavy object at A and describe the motion.

2. Now imagine a pendulum, with the length of the radius of the earth. Describe the motion of the pendulum. What is the period of the pendulum?

3. Finally, find out, or calculate the period of a satellite in circular motion, just above the surface of the earth. Compare the times for the three motions and comment.

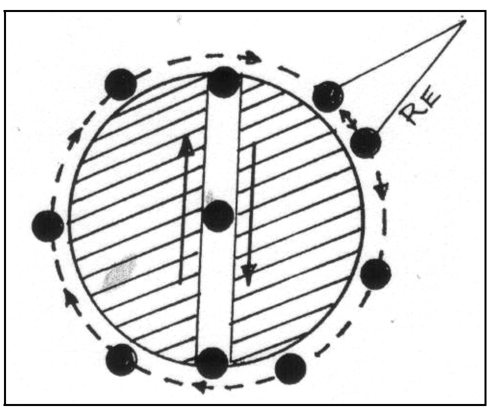


Fig. 44: Earth with a Tunnel Through It

Concluding Remarks

This LCP was guided by the history of the pendulum, based on the work of six philosophers and scientists on motion: Aristotle, Nicole Oresme, Galileo, Christian Huygens,

© 2007, Arthur Stinner

Newton, and Leon Foucault, from the fourth century BC to the middle of the 19th century. A dramatization was added, involving a number of short conversations with Aristotle, Oresme, Galileo, Huygens, Newton and Leon Foucault.

We have seen that the pendulum, together with the versatile inclined plane, played a central role in the development of the kinematics and dynamics in the seventeenth century. It was shown that in many of the key problems of Galileo these simple devices were connected in creative ways to study motion. We then described how Newton used pendula and Galilean kinematics to test and demonstrate his laws of motion.

Moreover, we have shown how the pendulum also played an important role in the next two centuries, mainly in the work of George Atwood and his famous machine to test Newton's second law of motion, and in the fascinating story of Jean Foucault' design of a very long and heavy pendulum to demonstrate for the first time directly that the Earth revolves around its axis. We concluded with the discussion of the thought experiment by the 14th century natural philosopher Oresme using our knowledge of Newtonian dynamics.

Having studied the laws of elementary kinematics and dynamics, we are now ready to really contextualize the study of physics, moving unto the study of scaling and modern robotics. We will see that the elementary physics of materials and of mechanics determine the limits of structures and the motion bodies are capable of. What is interesting is that the physical principles of strengths of materials goes back to Galileo, the dynamics of motion we need to apply is based on an elementary understanding of Newtonian mechanics, and the mathematics of scaling required depends only on an understanding of ratio and proportionality. Finally, the main ideas developed in LCP3 will be seen as intimately connected to architecture, biology, bionics, and robotics. It is hard to imagine a more motivating large context in which to teach the foundations of statics and dynamics with a strong link to the modern world around us.

IL 97 ** Discussion of gravity powered transportation—through earth tunnels
IL 98 *** Detailed discussion of gravity train—earth tunnel travel
IL 99 ** Brief discussion of gravity train—earth tunnel travel

© 2007, Arthur Stinner

REFERENCES

Cohen, B. (1981). Newton's Discovery of Gravity. *Scientific American*. March.

Metz**,** D. & Stinner, A. (2004). The Future of Physics Teaching. *The Alberta Journal of Science Education***.** *The Physics Teacher* . Vol. 40, November 2002, 487-492.

Matthews, M. (2000). *Time for Science Education: How Teaching the History of Science and Philosophy of Pendulum Motion can Contribute to Science Literacy.* Kluwer, New York

Stinner, A. and Metz, D. (2003). Pursuing the Ubiquitous Pendulum. *Physics Teacher*, Vol. 41.34-39.

Stinner, A. (2001). Linking "The Book of Nature" and "The Book of Science". *Science & Education*. **10:** 323-344.

Stinner, A. (1973). An Enrichment Program for Grade 13 Physics, *The Crucible* (The Journal of the Science Teachers' Association of Ontario) *4* (6), 22-30.

Stinner, A. (1994). The story of force: From Aristotle to Einstein. *Physics Education, 29*, 77-86.

Stinner, A. (1995). Contextual Settings, Science Stories, and Large Context Problems: Toward a More Humanistic Science Education. *Science Education, 79* (5), 555-581.

Tobin, W. (1998). Leon Foucault. *Scientific American.* July. pp. 70-77.

Michael Matthews, Colin Gauld & Arthur Stinner. (Eds). (2004). *The Pendulum: Scientific, Historical, Philosophical & Educational Perspectives*. Science & Education. Part I Vol. 13, Nos. 4-5. 2004.

More Excellent Applets:

IL 100 * Collection of Java applets

IL 101 **** Ten "most beautiful" experiments in physics—applets

IL 102 *** Comprehensive short history of scientific thinking, from Aristotle Newton

IL 103 **** Excellent for many physics applets

© 2007, Arthur Stinner

Conversation with Aristotle

(Back to History of the Pendulum)

(Back to Presentation of the Contexts)

Imagine that you could go back in time and have a conversation with great physicists. Our first scientist is Aristotle. We are in Greece, in the fourth century, BC.

Aristotle appears. He walks back and fourth in deep thought. The student approaches him.

Aristotle:

Greetings my young friend. In anticipation of your visit I have been thinking about motion around us. The motion of a leaf falling, a javelin being thrown, the motion of a cart, etc…Motion in general is an inexhaustibly deep subject. I understand that you wish to discuss motion with me.

Student:

Yes, I do. It is bewildering, I know. But you have clearly categorized motion. Especially, you have separated the laws of motion in the heavens and those that govern motion on Earth, *terrestrial* motion and *celestial* motion.

Aristotle:

Yes. But let us only talk about motion on Earth only.

Aristotle takes a stone and a feather. He looks at the student and then asks:

Which of these will fall to the ground first, the stone or the feather?

Student:

I think the stone will fall faster… .

Aristotle:

Alright.

He drops the stone and the feather. As expected, the stone reaches the ground first.

The student then takes a small piece of wood. He looks at Aristotle and says:

Student:

I have here a small piece of light wood. The stone is clearly many times heavier than the wood. Do you agree?

He gives these to Aristotle. Aristotle smiles and hands them back to the student. The student drops these and they seem to fall to the ground at the same time.

This simple demonstration suggests that all heavy objects fall at the same rate, don't you think?

Aristotle:

I am not convinced about that. I think if we went to a high tower and dropped these objects, then the stone would hit the ground first.

© 2007, Arthur Stinner

Student:

That may be so. However, you cannot say that if we had two objects, say two spheres made of different metals, but of the same size, and one twice as heavy as the other, the heavier one would fall twice as fast.

Aristotle:

Well, maybe not. But I have observed heavier than water objects fall through water."

He picks up two metallic spheres.

Let's place these in the water and allow them to fall.

He walks over to a glass container and lets the spheres fall through the water.

Now observe them as they fall through the water.

Student:

It looks like the heavier falls much faster.

Aristotle:

You see. Clearly, heavier objects fall faster in water.

Student:

Very impressive. But then, imagine an object falling toward the earth if there were no air. Don't you think that then a feather and a stone would reach the ground at the same time?

Aristotle:

No! That is impossible!

Student:

But why?

Aristotle:

Because if there were no air, we would have a vacuum above the earth. And… , I believe that vacuum cannot exist!

Student:

Aristotle, why do you believe a vacuum cannot exist?

Aristotle:

Because then any small force could make an object move at an infinite speed. And to have motion we need a force. Therefore, vacuum is impossible.

Student:

Yes, I can see that. But his is only true if we say 'the greater the push force the greater the speed' and also 'the less the resistance force the greater the speed for a given push force'.

© 2007, Arthur Stinner

Aristotle:

Exactly. The question then is: 'What is the cause of motion of a freely falling object or a projectile like a javelin after it is thrown?'.

Student:

According to your physics, first, 'for an object to move you need a force' and secondly, there are two kinds of forces, *natural* and *violent*. A natural force is the force that pulls objects toward the center of the earth and a violent force is a force like the one that propels a javelin.

Aritotle:

Very good. So, a freely falling object needs no explanation but the motion of a javelin does.

Student:

Where does the motive force for the javelin come from.?

Aristotle:

Again we have to invoke the principle that vacuum cannot exist. As the javelin moves through the air, the air behind the javelin rushes in to fill the momentary vacuum produced. This action of the air provides the pushing force.

Student:

But, surely, there must be an air resistance that opposes this force, Aristotle.

Aristotle:

Yes. But the push force of the air rushing in is greater.

Student:

I would be interested to know how you would explain this motion.

The student produces a pendulum and shows Aristotle how the attached mass moves back and forth.

Aristotle

He studies the motion and then smiles:

This is a very interesting motion. It seems to be a combination of free fall and constrained motion. Therefore I would say that it is partly natural and partly violent motion."

Student:

Thank you very much, Aristotle. This was very illuminating.

Aristotle:

You are welcome. You must come and join us in my Academy. Perhaps you can continue to study of the motion of a , what did you…, the pendulum.

Student:

Thank you, I will do so very soon. Good bye.

© 2007, Arthur Stinner

Conversation with Oresme

(Back to History of the Pendulum)

(Back to Presentation of the Contexts)

Our student meets Oresme in his room at La Saint Chapelle, in 1363, where he was elevated to the post of dean of the Cathedral of Rouen.

Oresme:

I am looking forward to our discussion, young man. I understand that you have studied the physics of Aristotle.

Student:

Master Oresme, thank you for agreeing to have this discussion with me. Yes, I have studied the physics of Aristotle, especially his ideas of motion.

Oresme:

Wonderful. He was a great thinker in every field of study, especially in logic and biology. But in physics, many of ideas were questionable. Especially those that had to do with motion.

Student:

Yes. I, too, was a little puzzled when Aristotle explains motion by way of his ideas of violent and natural motion. For example, his explanation of the cause of motion of a projectile is in my opinion, doubtful.

Oresme:

Actually, Aristotle's ideas about motion were first challenged about eight hundred years ago by John Philoponous (of?...) He rejected the Aristotelian law that the speed of an object depends directly on the force and inversely on the resistance. Specifically, he argued that speed is proportional to the force minus the resistance force.

Student:

That of course neatly sidesteps the problem of the vacuum.

Oresme:

Yes, indeed. I agree with Philoposus. That means, of course, that motion in a void (vacuum) where there is no resistance, is possible.

Student:

But we still have the problem of the "cause of motion".

Oresme:

Philoposus went further and argued that it is not the air that provides the motive power to propel a projectile like a javelin, but an impressed force that he called impetus. The impetus, however, dies out.

Student:

Is there any text available that was written by Philoposus?

© 2007, Arthur Stinner

Oresme:

Yes, indeed.

He picks up a heavy book and looks for a passage.

Here we are.

The concluding words in his *Commentary on Aristotle's Physics* sums it up well:

From these considerations and from many others we may see how impossible it is for forced motion to be caused in the way indicated. Rather is it necessary to assume that some incorporeal motive force is imparted by the projector to the projectile, and that the air set in motion contributes either nothing at all or else very little to this motion of the projectile. If, then, forced motion is produced as I have suggested, it is quite evident that if one imparts motion "contrary to nature" or forced motion to an arrow or a stone the same degree of motion will be produced much more readily in a void than in a plenum. And there will be no need of any agency external to the projector. . . .

Student:

He was certainly ahead of his time.

Oresme:

I agree. But my teacher, John Buridan, developed the impetus theory of Philiposus even further. He thought that an impressed force on a projectile was permanent unless acted on by resistances or other forces. I have been developing Buridan's ideas a little further. I believe that it is not possible to detect uniform straight-line motion.

Student:

Actually, that makes sense to me. When you are on a ship on a calm day, you are not aware of motion.

Oresme:

Exactly. This is why I believe that it may be the earth that is rotating and not the sun. But I have had problems with that idea because it is against the teachings of the Church. Remember, I am a Bishop in the Catholic Church.

He hesitates and then continues

Here is an interesting idea, my young friend. Imagine a tunnel through the earth. Let me show you.

He draws a sketch in the sand

Now imagine a heavy object being dropped into the tunnel. Also imagine that there is no resistance offered to the motion. What kind of motion would you expect?

Student:

Let me think. If we assumed the earth as a perfect sphere, that is, the material distribution were the same throughout, the object should arrive at the other end and stop. Then the motion should repeat itself. Why, the object would move like a pendulum.

© 2007, Arthur Stinner

Oresme:

Very good. Actually, this idea is not original with me.

Oresme opens a book and shows the student a passage.

My colleague Albert of Saxony discussed this idea first. Let me show you the text.

If the earth were completely perforated, and through that hole a heavy body were descending quite rapidly toward the center, then when the center of gravity (medium gravitatis) of the descending body was at the center of the world, that body would be moved on still further [beyond the center] in the other direction, i.e., toward the heavens, because of the impetus in it not yet corrupted. And, in so ascending, when the impetus would be spent, it would conversely descend. And in such a descent, it would again acquire unto itself a certain small impetus by which it would be moved again beyond the center. When this impetus was spent, it would descend again. And so it would be moved, oscillating (titubando) about the center until there no longer would be any such impetus in it, and then it would come to rest.

Student:

Wonderful. So now we have a better explanation of the motion of the pendulum than the one Aristotle gave us.

Oresme::

Indeed. The motion of the imaginary object falling through a tunnel then can be demonstrated by the motion of a pendulum.

Student:

But we still cannot make predictions about the time of oscillation involved for a given pendulum. For example, how many swings would a pendulum of length of 100 cubits make in given a time?

Oresme:

Maybe you will be able to do that when you become a natural philosopher.

I must go now. I hear the bells calling for vespers. Good bye.

© 2007, Arthur Stinner

Conversation with Galileo

(Back to History of the Pendulum)

(Back to Presentation of the Contexts)

Our time traveler finds himself in Galileo's laboratory in the year 1610. He looks around as Galileo suddenly appears. There is an inclined plane, pendula and many instruments on the table. One can see a water clock and a guitar. At the very front of the table there is a small telescope. Galileo is writing. He looks up.

Galileo:

Welcome to my laboratory, young man. I have just completed my manuscript *The Starry Messenger*, in which I describe the motion of the moons of Jupiter. A very revolutionary astronomical discovery. However, in your letter you said you are interested in the physics of terrestrial motion.

Student:

Well, if you have time we could also discuss your work on the motion in the heavens.

Galileo:

Well, we will see.

I am just working on an interesting problem.

He shows the student a pendulum, points to a very long inclined plane and a water clock.

I have shown that if a metallic sphere rolls down an inclined plane like this

He demonstrates the motion.

The distance covered is proportional to the square of the elapsed time.

So that if the sphere travels one cubit in one second, it will travel four cubits in two seconds, nine cubits in three seconds, etc.

Student:

How do you measure the elapsed time, Signor Galileo?

Galileo:

I have used my pulse at the beginning, but I found that a water clock gives the best result.

He points to a large cylinder

Student:

I see. So volume of water is equated with time.

Galileo:

Yes. ---

Student:

Did you just go ahead and measure distance traveled against time elapsed to arrive at this result?

© 2007, Arthur Stinner

Galileo:

Well, I could have done it that way. But that result would not have told me how speed and time are related. Can you see that?

Student:

I think so. But how many different ways could a ball roll as far as the relationship between time and speed is concerned?

Galileo:

At least three different ways. I called these my hypotheses about free fall.

Galileo stops for a moment and then continues.

Student:

But you are not measuring the free fall directly.

Galileo:

You see, I argued that I cannot measure the time elapsed in free fall directly, because of the small time interval involved….

He looks at the student..

Student:

Yes, I can see that. And then you extrapolate it to 90 degree vertical for free fall. Very imaginative.

Galileo:

Thank you. However, I could measure the elapsed time for a freely moving sphere along an inclined plane. I then argued that free fall motion was the same as the motion along the inclined plane. I simply "diluted gravity", as it were.

They both laugh.

Student:

Very clever, Signor!

Galileo:

Thank you. ---To come back to our problem. Try to guess these three hypotheses of motion, young man.

Student:

Let's see. We could guess that the speed of a freely falling object varies according to the distance covered.

Galileo:

Good. Unfortunately, this hypothesis leads to an absurdity. If you insisted, I could prove this. But let's ignore this one.

© 2007, Arthur Stinner

Student:

Thank you. The other hypothesis could be that motion is such that the speed varies according to time.

Galileo:

Very good. There is a third one but it turns out that it is really equivalent to this one.

He stops for a moment.

It is quite easy to show that if the speed varies as the time elapsed then the distance must vary as the square of the elapsed time.

Student:

I wish Aristotle and Oresme were here to hear this.

Galileo:

Well, Oresme would be impressed but I do not think Aristotle would be. I do not think he would have been interested in a quantitative measurement of motion.

Galileo walks over to the table and picks up an orange and grape.

But this may have baffled Aristotle.

He holds the orange about two cubits above the table and then drops then at the same time.

So, how would have Aristotle reacted to this simple demonstration, my young friend?

Student;

I don't know. He would probably have said that if you went to the top of a tall building and dropped these the orange would arrive first.

Galileo:

Well, we won't do that, even though the Leaning Tower is close by.---But let us return to the pendulum.

He makes the pendulum oscillate and looks at this motion, lost in deep thought.

As a young man, sitting in a cathedral during mass my thoughts drifted to other things. I noticed that identical chandeliers in a church swing with the same period, no matter what the amplitude is.

He explains the idea of a period, an amplitude and demonstrates it.

I also noticed that the mass of the chandelier doe not affect the period. I was convinced then and there that neither mass nor amplitude affected the period.

He smiles and then continues.

Of course, I used my pulse as a timing device.

© 2007, Arthur Stinner

Student:

That is very curious and unexpected. Actually, the discovery that mass makes no difference follows from the fact that all heavy masses fall at the same rate. But that the amplitude does not effect the period I find astonishing.

Galileo:

Yes, so did I. Next I wanted to find out how one can calculate the period of a pendulum on the basis of the length alone.

Student:

I would have guessed that if you doubled the length, the period would also double.

But that must be wrong.

Galileo:

Yes, it is. ---I first argued using geometry and free fall-motion that the time it takes for a sphere to roll down any inclined plane that connects from the base point of a circle is the same as it would take a freely falling object to fall the distance equal to the diameter of the circle.

Galileo goes to the blackboard draws the appropriate diagram and explains.

Student:

This, too, is not obvious, Signor Galileo. But how is this connected to the period of the pendulum?

Galileo:

Well, it suggests that the period of the pendulum is proportional to the square root of the length. We can easily show this.

Galileo counts the time it takes for ten swings for a certain length. Then he doubles the length of the pendulum and repeats the experiment.

Galileo looks at the student

What length do we need to double the period?

Student:

That is easy. Four times the original length!

Galileo:

Bravo!

Student:

Thank you. Finally, I would like to have your opinion on something that Oresme suggested. Almost three hundred years ago he said:

A body falling through a well that has been drilled from one side of the earth, through the center, to the other end, would oscillate like a pendulum and eventually come to a stop at the center.

© 2007, Arthur Stinner

Galileo:

Yes. I, too, discussed this idea in my book

He brings a book and opens it.

… From this it seems possible to me... to believe that if the terrestrial globe were perforated through the center, a cannon ball descending through the hole would have acquired at the center such an impetus from its speed that it would pass beyond the center and be driven upward through as much space as it had fallen, its velocity beyond the center always diminishing with losses equal to the increments acquired in the descent; and I believe that the time consumed in this second ascending motion would be equal to its time of descent.

Student:

Very interesting, Signor Galileo. This is very similar to the statement made by Albert of Saxony and then later endorsed by Oresme, almost three hundred years ago. Were you aware of these?

Galileo:

I really cannot remember. I probably was.

Student:

When you compare your statement with the one written by Albert of Saxony, you don't seem to mention the idea of impetus.

Galileo:

I have been thinking about the "cause of motion" rather than the description of motion. Therefore, I tried to understand how "impetus" could be quantified and measured. For example, I have done experiments with heavy objects falling into soft soil. I have found that if you double the speed of impact you will get four times the penetration effect. I called this quantity *velocitas,* which should not be confused with the velocity.

Student:

So you think that impetus has to do with a quantity you call *velocitas*, that is measured by the square of the velocity?

Galileo:

Yes. Clearly the force applied to an object accelerates that object. In this manner the force of gravity acting on a mass accelerates that object with an acceleration of free fall. What is mysterious is the fact that the acceleration is always the same for all heavy objects.

Student:

I see. Let me clearly understand what you are saying, Signor Galileo.

The student moves to one of the tables and lifts up a toy cart, places it back onto the surface and pulls on the string attached to it.

If I pull a small cart with a given force, it will accelerate at a certain rate and if I doubled that force it would accelerate twice as much. So that if I applied the force for one second in both cases the cart would reach a velocity of twice this value for force which was doubled.

© 2007, Arthur Stinner

Galileo:

Very good. ----But now compare the distances moved and the velocities reached by the cart for the two cases.

Student:

Well, according to your formula that distance traveled for a constantly accelerating object is proportional to the square of the velocity. Therefore, the distance traveled for the second case is four times as much.

Galileo:

Exactly! So it seems that there is a quantity associated with the square of the velocity and the distance through which an object moves while a force is acting on it that will be important in the new dynamics. But I will leave this development to younger men, like yourself.

Student:

Thank you, Sir

Galileo:

Galileo walks back to the pendulum and slowly pulls the sphere away from its rest position.

Let's go back to the problem of an object falling through the Earth. You can feel that the force I require to pull the pendulum increases as I pull. That means that the acceleration itself must change.

The student tests this and nods.

Galileo:

Now things become interesting. We have a change of a change, that is, a change of acceleration is a change of a change of velocity.

He stops and thinks.

Student:

But how do we know that the change of acceleration is a uniform change?

Galileo:

Exactly! Even though I know that the period is proportional to the square root of the length of the pendulum, I do not know the exact motion of the mass as it moves.

Student:

But you know that the maximum velocity of a swinging pendulum will be at the lowest point.

Galileo:

Yes. And you also know that the oscillation will eventually stop. Again, it will be a future physicist that understands the dynamics of motion better than I who will be able to describe the motion exactly.

© 2007, Arthur Stinner

Student:

Maybe we can conclude by comparing the motion of a heavy object falling through the earth with the motion of a pendulum.

Galileo:

Good. We could argue that, as the object falls through the tunnel the force of gravity diminishes in the same way the force acting on a pendulum does.

Student:

I can see that. An obvious question would be: What length of a pendulum would produce a period equal to that of our object falling through the earth?

Galileo:

An interesting question. Let me see. Maybe the answer is that the length should be equal to that of the radius of the earth?

Student:

Using your formula we could figure that out by comparing to standard length that gives us a period of one second.

Galileo:

Yes. Well, I will let you work that out.

He hesitates and then continues.

But, young man, I would like to show you something that is not obvious.

He walks to the pendulum and pulls it out to a large amplitude.

Remember when we showed earlier that a ball will roll down any inclined plane that connects the center of a large circle and a point on the circle in equal times?

Student:

Yes, I do. A very surprising result.

Galileo:

Well, I can show you something even more surprising. Imagine an inclined plane here, just as in our sketch earlier.

He points to the earlier sketch.

Now if I release the pendulum and the ball on the inclined plane at the same time, which one will arrive at the bottom first?

Student:

Well, common sense would tell me that the ball on the inclined plane will arrive first.

He hesitates and then continues.

But since you said it was surprising this must be wrong.

© 2007, Arthur Stinner

They both laugh.

Galileo:

Let's see. Come and help me with this so that we have the balls beginning to roll at the same time.

The student assists Galileo and the balls are released at the same time.

Student:

Amazing. But we can actually compare the times of descent to predict this.

Galileo:

Yes. I will leave this with you as home work. I must go now. I have an audience with one of the cardinals. He wants to look through my new telescope.

Student:

Do you think he will have the courage to look through your telescope?

Galileo:

We will see.

© 2007, Arthur Stinner

Conversation with Huygens:

(Back to History of the Pendulum)

(Back to Presentation of the Contexts)

Our student finds himself in the laboratory of the Dutch physicist and mathematician Christian Huygens, sometime in 1687. Newton's *Principia* was just published, but we assume that Huygens has not had time to study it.

The most important of Christian Huygens' written works was his *Horologium Oscillatorium* published in Paris in 1673. It discussed the mathematics surrounding pendulum motion and the law of centrifugal force for uniform circular motion.

The most obvious things noticed are a small billiard table, clocks, springs and several pendula. Huygens welcomes him.

Huygens:

Welcome, my young friend. What shall we talk about?

Student:

Thank you, Dr. Huygens.

The student looks around and smiles.

Do you play billiards?

Huygens:

Sometimes. But the reason for the billiard table in my laboratory is for experimenting with collision. I believe that we can test our ideas about dynamics best by studying the physics of impact and collision between billiard balls.

Student:

Later perhaps I want to know about these experiments. But now I am interested in finding out about your experiments and theory of the motion of the pendulum. I especially want to know what you have added to the findings of the great Galileo.

Huygens:

All right. Of course, Galileo has shown us many properties of this wonderful phenomenon we call oscillatory motion. But I have found new ones as well as shown that Galileo was wrong in some instances.

Student:

I have always thought that Galileo's insistence that the period of the pendulum is independent of amplitude must be wrong. Even in his time, the mathematician Mersenne claimed that he had experimental evidence to believe that the period changes significantly for angles over 20 degrees.

Huygens:

I can easily show you that Galileo was wrong.

He has a long pendulum oscillate for 10 swings using an amplitude of about 10 degrees. Then he increases the amplitude to about 30 degrees. The times are significantly different.

© 2007, Arthur Stinner

Student:

Very convincing. Surely, Galileo must have checked this.

Huygens:

Of course he did. But he believed so strongly in the isochronous property of the pendulum that he probably explained away the difference by suggesting that frictional effects contributed to the difference.

Student:

That makes sense.

Huygens:

It is interesting that Galileo also rejected Kepler's work in astronomy. He always believed that the motion of planets must be circular, despite strong evidence for the motion of Mars to be elliptical.

He stops for a moment

But let's continue. As we have already mentioned, Galileo believed that the motion along an arc of a circle is isochronous. But I have shown that it is the cycloid that guarantees isochronous motion. Let me show you.

Huygens shows a pendulum whose motion is guided along a cycloid. Makes the pendulum oscillate. He measures the time take for 10 swings for different amplitudes

Student:

Very convincing. Could you then use this principle to make clocks that are more accurate? Sailors could use a good clock that keeps accurate time over long period to establish the longitude of their position.

Huygens:

Yes, indeed. Latitude is easily measured by simply finding the angle to Polaris but to measure longitude requires a complicated astronomical observation, using the Moon or the moons of Jupiter.

Student:

Yes. I have studied this a little. I believe that if you had a reliable clock that is accurate to a few seconds a day you could then find your latitude, to a good approximation, degrees measured East or West of Greenwich I believe, by observing the time of passage of the midday sun.

Huygens:

Very good. But we still cannot do this. I have built a marine clock as early as twenty years ago. But it did not survive rough seas. I am personally convinced that it will not be a pendulum driven clock to accomplish this but a mechanical-spring driven clock. I had some success with such a device that I built over 20 years ago. But I do not think this will happen in my lifetime.

© 2007, Arthur Stinner

Let us continue with the pendulum. I have studied the conical pendulum and using my formula for centrifugal force been able to find a formula for the period of the pendulum that is superior to the one that Galileo found.

Student:

Galileo showed that the period of the pendulum is proportional to the square root of the length.

Huygens:

Yes. But he was not able show that the strength of gravity varies on the surface of the Earth. In my formula the gravity is part of the expression. That means that I am able to find the acceleration due to gravity for various places on Earth.

Student:

Can you show me please?

Huygens:

I would be glad to explain it and show you with a simple demonstration.

He goes to the table and picks up a pendulum. He rotates the ball in a circle.

I can show that the rotation period for a small angle is the same as the period of oscillation of a pendulum with a length equal to the vertical distance.

He draws a picture on the black board and explains and derives the formula for the pendulum.

Using the expression $\mathbf{T = 2\pi\sqrt{(l/g)}}$ I can now find the value of gravity in terms of free fall.

He stops for a moment band then continues.

But that is not how we compared the gravity of various locations on Earth.

We determined the strength of gravity by finding the length of a one-second pendulum for a given location.

Student:

But how do you determine the value of this length for various places.? You must have standard against which you measure.

Huygens:

Yes, Indeed. In 1672 the astronomer Richer found that the length of the seconds pendulum that was calibrated in Paris was shorter on the equator than in Paris, as measured in Cayenne, South America. The values found were about 3/100 different.

Student:

This difference must be due to the centrifugal force produced by the rotation of the Earth.

Huygens:

Yes, exactly.

But you must remember that an ordinary simple pendulum like this one is not good enough to detect the difference for the value of g for various locations.

© 2007, Arthur Stinner

He walks to his table and picks up an apparatus that looks complex but resembles a modified pendulum.

This is a compound pendulum where the center of mass can be mechanically adjusted. This pendulum can be seen as a research instrument to determine the value of gravity with a high accuracy and precision.

Student:

What are you working on right now, Mr. Huygens?

Huygens:

Well. First of all, I must look at Isaac Newton's *Philosphiae Naturalis Principia Mathematica,* which has just been published. It is written in Latin so that all scholars can read it. I am sure this work will occupy me for some time.

He smiles and then continues

Perhaps you can back in a few months to discuss the *Principia,*

Student:

Yes, that would be nice.

© 2007, Arthur Stinner

Conversation with Newton

(Back to History of the Pendulum)

(Back to Presentation of the Contexts)

Our student finds himself in Newton's Laboratory. There are instruments, chemical apparatus (alchemical), a globe, several large prisms, as mall reflecting telescope and pendula. Newton is working on a manuscript, he is indicating with his hand that the student sit down. It is summer time in the year 1712. Newton has been Master of the Mint since 1699, was elected president of the Royal Society, after the death of Robert Hooke.

The Royal Society commission , under Newton's direction, investigates the competing claims of Leibniz and Newton to having developed the calculus. The commission decides in favor of Newton.

Newton:

Sit down please. Ordinarily I do not give interviews. They are a waste of time for me. But you come highly recommended as someone who has studied the history of natural science deeply, especially the physics of motion.

Well, we will see.

He gets up and moves to a chair close to his guest.. He points to his desk.

Young man, do you recognize the items on my desk?

Student:

Yes, I do, Sir Isaac. I recognize the prisms, the telescope, the globe, and especially the pendula. I have studied your Optiks and have tried to work my way through the less intimidating parts of your *Principia*.

Newton nods with approval.

I know that you constructed a reflecting telescope as a young man after your studies of optical phenomena. You argued that refracting telescopes were subject to color interference and therefore argued that they were outdated. I also recognize some containers that suggest that you are doing chemical experiments.

Newton:

Very good. But why are you especially interested in pendula?

Student:

Well, I have been following the use of pendula in understanding motion. Aristotle was puzzled by the motion of a pendulum, Oresme used pendula to illustrate motion, Galileo found them very useful in his studies of free fall and used them as a timing device. Recently Huygens investigated the motion of pendula in his *Horologium Oscillatorium.* I believe that pendula were very important in your work in discovering the laws of motion.

Newton:

Yes, indeed. Well, I am impressed by your general knowledge of natural philosophy. Let me see.

He gets up and paces around, stopping to pick up a pendulum.

© 2007, Arthur Stinner

<u>Student</u>:

It is puzzling to me that you were able, by yourself, to find the underlying laws of motion.

<u>Newton:</u>

The laws of motion did not come from divine revelation.

They both laugh.

Many people seem to believe that they were self-evident to me, or came full-blown to the mind of the great Newton, shortly after an apple fell on his head.

Again, Newton laughs.

To come back to your original question. I think I have been able to do much because I keep the subject constantly before me and wait until the first dawnings open slowly, little by little, into full and clear light.

He looks reflective and then continues.

But let us go back to pendula. I have used pendula extensively for the confirmation of my laws of motion. Let's start with the first law. Perhaps you can state this law.

<u>Student:</u>

Yes., I can. I memorized your laws straight from the *Principia*. But, of course, that does not mean I understand them.

Newton smiles and nods.

Why don't I state all three of them?

<u>Newton:</u>

All right then. You can state them first and then we can discuss them.

<u>Student:</u>

The first law states that

An object at rest or traveling in uniform motion will remain at rest or traveling in uniform motion unless acted upon by a net force.

The second law says that:

The rate of change of momentum of a body is equal to the resultant force acting on the body and is in the same direction.

The third law makes a very general statement:

For every action there is an equal and opposite reaction.

<u>Newton:</u>

Good. But as you said just because you can recite them does not mean that you have a good understanding of them.

Let us look at them one by one.

© 2007, Arthur Stinner

The first law is really just a statement of Galileo's law of inertia. Galileo stated this law and his mental picture was an object moving on a perfectly frictionless surface of the Earth. So he saw the motion that can be thought of as 'the unimpeded circumnavigation of the Earth'.

Student:

Yes. I can see that.

Newton:

But my mental picture is this: "motion in deep space where gravity is negligibly low. That is 'unimpeded constant speed in a straight line in deep space'.

Student:

How can this law be confirmed?

Newton:

Well, it really can't be experimentally confirmed. One can only make a mental picture of it. You could call this a "thought experiment".

Now the second law can be illustrated by analyzing the motion of a pendulum.

He goes to a long pendulum and attaches a spring to the hook that is seen in the large wooden sphere. Newton pulls the sphere a little to a certain small displacement and then pulls it to about twice the original displacement.

My friend Robert Hooke showed (before the *Principia* was written) that when a spring is pulled, the extension of the spring is proportional to the force applied. This law applies here.

Student:

I see. A pendulum oscillates like a spring.

Newton:

Exactly.

He demonstrates this with a pendulum and a spring oscillating with the same period.

Now I can use the second law and show that the period of a pendulum is given by the very formula that Huygens derived about thirty years ago. He did this by analyzing the motion of a conical pendulum and using the formula for centrifugal acceleration that he found.

Newton stops for a few seconds.

You should notice that Huygens did not use dynamical laws, he only used kinematics.

Newton suddenly seems to remember.

Actually when he derived his expression for centrifugal acceleration he did mention the idea of force.

Student:

But does Hooke's law apply for all displacements?

© 2007, Arthur Stinner

Newton:

Unfortunately, no. You can easily show that after about 10 degrees of displacement from the vertical we will have problems. In fact, as Huygens showed, the period of a pendulum becomes larger as you increase the amplitude. You would find that the period of a pendulum for a displacement of 90 degrees is about 17/100 longer than the period for small displacements..

Student:

Clearly this is why Huygens had to find the curvature of motion that produced a tautochronous motion, that is motion such that the time of oscillation is constant for all displacements. He showed that a cycloid was that curve.

Newton:

Exactly. Galileo believed that along the arc of a circle a sphere would roll down to the lowest point in the shortest time and that it also provided the curve for a tautochronous motion.

Student:

So he was wrong on both counts.

Newton:

Yes. And it is interesting that Huygens did not realize that the cycloid is also the curve along which a ball descent in the shortest time.

Student:

This gives me a chance to ask you about the challenge that the famous Swiss mathematician Johann Bernoulli sent to all the leading mathematicians of Europe, 1697,

I believe.

Newton:

Oh, yes. The challenge was, something like:

Given two points A and B in a vertical plane, what is the curve traced out by a point acted on only by gravity, which starts at A and reaches B in the shortest time.

I remember (I think it was in 1696). I cam home from the Mint, in the midst of the hurry of the great recoinage, did not come home till four in the afternoon from the Tower very much tired, but did not sleep till I had solved it, which was by four in the morning.

Student:

That is an incredible story of the power of concentration on a difficult problem, Sir Isaac.

Newton:

Well, you know, I do not love to be pestered and teased by foreigners about mathematical things...

Student:

I have heard the story that when Johann Bernoulli looked at your proof, after it had been published anonymously, he said: “ I recognize the lion by his print”.

© 2007, Arthur Stinner

Newton smiles.

Student:

Your solution of the problem was based on geometric reasoning only, whereas Leibniz, and the Bernoullis solved it by the analytical method using algebra and the Calculus.

Newton:

Well, you know algebra is for bunglers.

He smiles and then continues.

But in a more serious vein, the challenge of the problem implied that it could only be solved by those who knew the secrets of Leibnizian calculus. I wanted to show them that geometry is still king in computing such things.

But, my young friend, we have strayed from the discussion of the three laws of motion.

Student:

Yes. Well, the third law seems to be the most general. The simplest form of the law is:

For every action there is an equal and opposite reaction.

Newton:

I think this is the law that most people seem to intuitively understand,--at least most can state it.

Student:

But it is not clear how to interpret it when considering dynamics. How do you understand this law?

For example, when I push against a cart the force the cart "feels" is the same force that I feel, except that it is in the opposite direction.

Newton:

Good. This is so no matter how you move the cart. Now if you look in the *Principia,* volume 1, you will find a sketch of colliding pendula.

He picks up two very long wooden pendula, one about twice as large as the other.

I experimented with the collision of these two pendula. What I found was that the motion before collision, as measured by the quantity of mass times velocity was the same as it was after collision. In other words the quantity mass times velocity is indestructible, as it were.

Student:

This result is certainly not obviously contained in the general statement of the law.

Sir Isaac, could you briefly summarize the role pendula played in establishing these three laws of motion?

© 2007, Arthur Stinner

<u>Newton:</u>

Good. Very few people understand that I struggled to establish force as a unifying concept. As a young man I believed in the impetus theory of Oresme. I also believed in the idea of transfer, the idea that one body may give up some of its force to another during impact.

<u>Student.:</u>

I also believed these things before I studied the *Principia*, Seems like I am in good company.

<u>Newton:</u>

Yes, you are.

Eventually I understood the idea of inertial mass and now it was possible for me to think of motion without force. I first investigated the idea of force as far Galileo's law of free fall is concerned.

He says emphatically:

But the notion of force had to be reconciled with how it was used in two other senses.

He looks at the student and asks:

Can you think of these two other senses of force in dynamics?

<u>Student:</u>

Well, I suppose the force between two colliding wooden balls would be another sense of force.

<u>Newton:</u>

Very good. And the third?

<u>Student:</u>

I can't think of it.

<u>Newton:</u>

Let me give you a hint.

He walks to the table and pick up one of the pendula. He then makes the wooden ball circle.

<u>Student:</u>

Of course, centrifugal force!

<u>Newton:</u>

I had great difficulty in getting rid of the idea of centrifugal (center-fleeing) force in describing the force on a body in circular motion. It was difficult for me liberate myself from this idea, believed by Descartes as well as Huygens. Finally, I understood that the force in the case of a conical pendulum must be understood as a center-seeking or centripetal force.

<u>Student:</u>

I see. So when we consider the revolution of the moon around the earth, what produces this centripetal force is the gravity between the two bodies.

© 2007, Arthur Stinner

Newton:

Exactly. And remember, the gravitational force of the Moon on the Earth is the same force as the gravitational force of the Earth on the Moon, but in opposite directions.

Student:

Yes. Could you summarize these forces?

Newton:

Yes. For linear motion we have force in terms of rate of change of momentum given my $\mathbf{m\Delta v / \Delta t}$, as demonstrated by the oscillation of a pendulum; for collision we have the quantity $\mathbf{m\Delta v}$, as demonstrated by the collision of two pendula, and for centripetal acceleration we have the quantity $\mathbf{m\, v^2 / r}$, as demonstrated by the conical pendulum.

Student:

Thank you, Sir Isaac. I have learned a lot today.

But I have a last question.

Newton:

Please. There is some time left, before I have to go to the Mint.

Student:

Keeping with the ubiquitous pendulum, I remember that Nicole Oresme in the fourteenth century suggested that if we could dig a tunnel through the earth an object falling through this tunnel would oscillate like a pendulum. Later, Galileo investigated this, what we may call a mental experiment. It seems to me we could use your laws of motion to investigate this problem.

Newton:

Actually I have thought about this problem some time ago.

He goes to the blackboard and makes a sketch of the Earth, a tunnel through it and a mass falling through it. He also draws a picture of a mountain with a cannon.

You see, the force on the mass falling through the Earth decreases linearly because it is governed by Hooke's law. Do you know why this is so?

Student:

I think so. In the *Principi*a you showed that if you descended into the Earth with a constant density the gravitational effect on the descending mass would be due only to the mass of the Earth underneath it.

Newton:

Very good. That condition guarantees that the gravitational force acting on the descending mass is linear.

Student:

I see, Therefore the motion must be an oscillation like that of a mass on a pendulum.

© 2007, Arthur Stinner

Newton:

But there is more. The period of a cannon ball being shot out from a cannon high above the Earth (I am assuming no friction due to the atmosphere) will be the same. What is more, an imaginary pendulum that has a length of the radius of the Earth would also have the same mass.

Student:

Incredible, Sir Isaac.

But I have a final question: What would be the period of an object falling through a tunnel that connects any two points on the surface of the Earth?

Newton:

That would be a good question for you to answer. I think I know the answer.

But I must go now to the Mint. See you in a few days.

© 2007, Arthur Stinner

Conversation with Foucault

(Back to History of the Pendulum)

(Back to Presentation of the Contexts)

We are in the laboratory of the French physicist and engineer Loen Foucault. It is a summer day in 1851. Just a few months ago in the spring Parisians flocked to Foucault's 67 m long pendulum in the dome of the Pantheon. The pendulum swung for about six hours and the swing plane had veered, as expected, about 70 degrees clockwise, showing that the Earth was revolving counterclockwise.

The experiment caused a sensation and Foucault became an instant international Celebrity. His lab is cluttered with instruments of all kind. We see his famous rotating mirror that he used to show that the speed of light is lower in water than in air. A large lathe is in the middle of the table and beside it we see what looks like a copper sphere. At the edge of the able we see mount with a gyroscope. There is a small reflecting telescope that Dr. Foucault built in preparation for the large reflecting mirror telescope he built for the astronomer Le Verier who used it for the discovery of Neptune. Dr. Foucault is seen adjusting the gyroscope. He looks up and points to a chair for his guest to sit down.

Foucault:

Please sit down, young man. I am just making a final adjustment on this instrument. I will be right with you.

His guest looks around with fascination. Foucault leaves the table, comes over to his guest and shakes his hand.

Student:

Thank you, Dr. Foucault, for allowing me into your lab. I realize that you have done important work in photography as well as in physics. I have just read your publication about how you and the physicist Fizeau have experimentally confirmed that the speed of light is lower in water than in air.

The argument

Foucault;

Thank you. That was a nice experiment. But do not believe that all physicists now believe that light is a wave phenomenon. It will take some time. Newton's dominating authority (as you know he defended the particle theory of light) is still with us.

Student:

Yes. I am acquainted with the dispute between Newton, defending his particle theory of light, and Hooke and Huygens who argued for the wave theory.

Foucault:

Well, as far as Fizeau and I are concerned, this was the crucial experiment that should settle The great physicist Arago, whose idea the experiment actually was, also believes that we have established the nature of light as a wave phenomenon.

He stops and looks sad.

© 2007, Arthur Stinner

Unfortunately, Arago is now blind and was unable to see the actual procedure.

Student:

That is sad.

But, Dr. Foucault, I would like to talk to you about the role of the pendulum in physics in general and in your recent dramatic demonstration of the rotating Earth in the Pantheon.

Foucault:

The pendulum has been an important, if not an indispensable tool, for developing ideas about motion in general, from Galileo to Huygens to Newton

Student:

And now to Foucault:

Foucault:

Thank you. But I don’t think I can be mentioned along with those giants of physics.

Student:

I disagree.

How did you get the idea for constructing your now famous pendulum? I think this is one of those demonstrations, which, in retrospect, seems obvious.

Foucault:

I agree.

The idea of finding a direct and publicly understandable demonstration to confirm the rotation of the Earth around its axis was always in the back of my mind.

Student:

Since the time of Galileo people have been aware

And now to the direct demonstration that the Earth rotates.

Foucault:

I was inspired to do this experiment when, working in my lab, I happened to twang a steel rod that was clamped in the chuck of a lathe. I noticed that although the rod rotated with the chuck , the plane of vibration remained the same.

He goes to the table and the lathe He demonstrates this effect.

Student:

Wonderful. I find it absolutely surprising.

Foucault:

Yes. Everyone, without exception, finds it so.

© 2007, Arthur Stinner

<u>Foucault:</u>

Interesting is that once you see it, you can then explain it. But it would have been difficult to predict it.

Well, young man, why do you think is the explanation?

<u>Student:</u>

The explanation must be in Newton's first law: "inertia keeps objects in the same state of motion unless they are they are disturbed by an external force.

<u>Foucault:</u>

Very good. I think Galileo and Newton would have had the same inspiration had they seen this phenomenon.

<u>Student:</u>

Yes. It is puzzling that no one before you tried this demonstration.

<u>Foucault:</u>

But let me continue with my story.

I was convinced that if I had a very long and heavy pendulum I could demonstrate that the pendulum would veer slowly in a counterclockwise direction. Thus the clockwise rotation of the Earth would manifest itself to an observer on Earth, in Paris, for example.

<u>Student:</u>

This sudden insight that you gained immediately after looking at the rotating lace and the twanged rod could be compared to the sudden insight of the young Galileo when he noticed that chandeliers of different swing had the same period of oscillation.

<u>Foucault:</u>

The difference of course is that he made a crude measurement right in the cathedral.

<u>Student:</u>

Oh yes. He used his pulse. But you could not and did not need a measurement.

<u>Foucault:</u>

True. Well, as you can imagine I started working on constructing such a pendulum immediately. I went to the cellar of my mother's house and there I observed the swing of a pendulum that was two meters long and had a brass ball of about 5 kg..

<u>Student:</u>

Did a short pendulum like that veer that could be noticed?

<u>Foucault:</u>

Well, first of all, the wire snapped at first try. But I managed to construct another pendulum several days later. I could clearly identify a swinging plane.

© 2007, Arthur Stinner

Later the physicist Arago urged me to build a longer one and I managed to study the swing of an 11 m pendulum at the Paris Observatory.

Student:

When did you have a public demonstration?

Foucault:

I first was allowed to make an announcement at the Academy of Sciences. There I made the assertion that the pendulum's swing plane would seem to veer by 360 degrees a day on the North Pole but elsewhere it would veer slower, that is inversely according to the sine of the latitude of the location.

Student:

I see. That means that on the equator there would be no veering.

Foucault:

Exactly. It is then easily predictable that in Paris the pendulum would rotate counterclockwise 360 degree in about 32 hours.

Student:

I see. But were you able to have the pendulum oscillate that long?

Foucault:

Actually. When we demonstrated this phenomenon to the public in the spring of 1851, we had a pendulum 67 m long and a mass of 28 kg.

Student:

Amazing.

Foucault:

Indeed. We managed to have this giant pendulum swing for almost 7 hours. It veered by about 70 degrees. This was expected.

Of course air resistance would slow down the swing.

Student:

Yes. I understand that this experiment was soon repeated around the world.

Foucault:

Yes. In Rio de Janeiro, which is south of the equator, the swing plane moved counter clockwise, as expected.

He moves to the table and picks up a pendulum.

Student:,,,**To be completed…**

© 2007, Arthur Stinner