LCP 5: Newton's Dream–Artificial Satellites: From Sputnik to the Space Shuttle

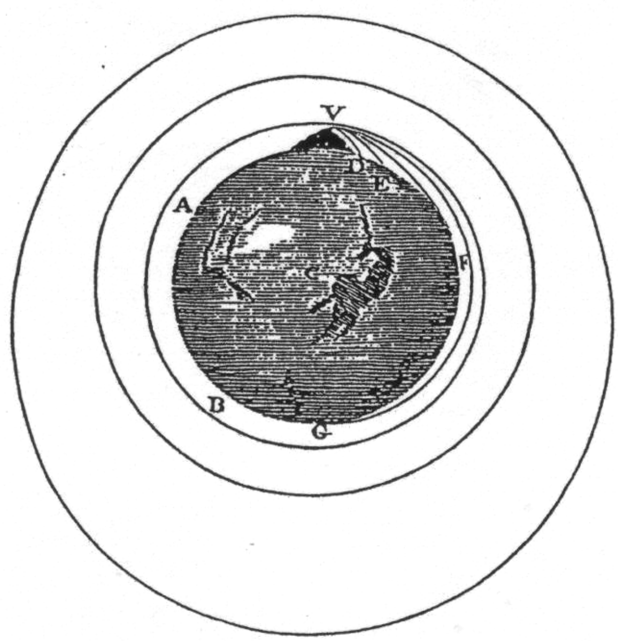


Fig. 1: Newton's Cannon

IL 1 **** An excellent IA using Newton's cannon

THE MAIN IDEA

In LCP 1 we discussed Newton's ideas of placing a cannon ball in orbit. This exercise was placed in his *Principia* with a sketch that Newton himself provided (See Fig.1). You should go back to LCP 1 and study this section again.

The following contemporary introduction to satellite motion would be completely clear to Newton.

Astronauts in space stations feel "weightless", while orbiting the Earth just like the astronauts in the Shuttle do. On earth you feel "weightless" when, for example, you jump off a diving board. There is gravity, of course, but the effect on your body is equivalent to being in zero-gravity. Similarly, for the person that is rotating or orbiting around the Earth in the space station the effect is the same as if there were no gravity, even though the gravity may be as high as .9g. That is so because the person is really falling freely toward the earth, i.e. vertical free fall due to gravity ($h = 1/2gt^2$) is just balanced by the horizontal motion due to inertia ($d = v\ t$, according to Newton's first law of motion).

If you want to feel a force in your frame of reference (which is freely falling relative to the earth), you produce a motion that does not interfere with the rotation around the earth but can still bring about a fictitious force of the right magnitude and direction. Spinning your frame of reference would be such a motion. Now, if you spin your frame of reference just right you can bring about a centrally directed force (centripetal force) of the same magnitude as mg. Thus, using $a = g = v^2/r$ you can determine the rotation necessary to obtain an "artificial" gravity. You can review the discussion in more detail about centripetal motion in LCP1 and 2.

THE PRESENTATION OF THE CONTEXT

The Physics of Orbits

In Newton's thought experiment a cannon was placed at a height of about 40 km. He showed that only one specific speed (launched tangentially) would produce a circular orbit. Of course, we don't launch satellite that close to the earth's surface because the atmosphere, even though it has a very low density at this height, would soon slow down the satellite and the satellite then would spiral into the denser lower layers and burn up.

IL 2 *** History of artificial satellites

The first artificial satellite, dubbed Sputnik, was launched by Russian engineers and scientists in 1957. The satellite was spherical, a 184-pound (84kg) capsule. It achieved an Earth orbit with an apogee (farthest point from Earth) of 584 miles (940 km) and a perigee (nearest point) of 143 miles (230 km), circling the Earth every 96 minutes. See IL 3 below.

We will begin by verifying the above, using a *guided problem solving approach*, and then set a simpler problem where the orbit is circular. The problem of elliptical orbits will be discussed in detail in LCP 7, "Journey to Mars".

IL 3 *** Details of the Sputnik Orbit

1. **Verifying the Data Above**

According to the data above, Sputnik 1 was in a highly eccentric orbit. To find the period of orbiting, we can use Kepler's third law, which states that *the ratio of the period squared and average radius cubed for a satellite is a constant.* This can be written as

$$\mathbf{T^2 / R^3 = Constant\ (K_E)}.$$

To calculate the period of Sputnik, we first determine the value of the constant for the Earth/satellite system. We already have a satellite in the sky, namely the moon. Newton used the moon and Kepler's third law to confirm the inverse square law of gravity. We will use the moon's period and the average distance from the centre of the earth to calculate the constant of the earth/satellite system. We should note, however, that the constant established this way will not be very accurate, because we will neglect the mass of the moon.

Use the following data to calculate this constant:

Period of the moon: **27.3 d.**

The mean distance between the earth and the moon: **3.84×10^{8} m**

a. Show that $\mathbf{K_E = 9.84 \times 10^{-14}\ (s^2 / m^3)}$

b. Now calculate the period of Sputnik 1 and show that it is about 96 minutes.

2. Calculating the Period of a Satellite in Circular Orbit.

Imagine that Sputnik 1 was placed in a near- circular orbit at a height of about 250 km instead of an elliptical orbit. The speed of Sputnik was about 28,000 km/h or 7800 m/s. See IL 3 above.

Fig. 2: The First Artificial Satellite, the Russian Sputnik, 1957

a. Calculate the gravity at a height of 250 km. Show that this value is about 9.1 m/s^2, or about 0.93 g.

b. Show that the speed of Sputnik (7800 m/s) at a height of 250 km, produced a centripetal acceleration of about .93 g.

c. Calculate the period of the satellite. Show that it is about 88 minutes.

The launching of the first artificial satellite by Russia caught most Americans by surprise. See newspaper article shown in IL 4.

IL 4 *** Contemporary report on the launching of Sputnik

Our movies and television programs in the fifties were full of the idea of going into space. What came as a surprise was that it was the Soviet Union that launched the first satellite. It is hard to recall the atmosphere of the time. (John Logsdon)

The Launching of a Satellite

IL 5 *** History of satellite launching

The following is taken from LCP1, and placed here as a review

IL 6 *** Complete description of orbits, including historical and mathematical background

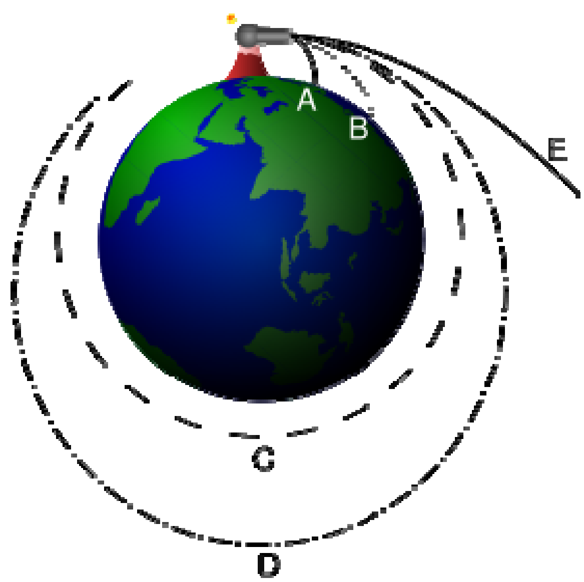


Fig. 3: A Picture Based on Newton's Sketch in the *Principia*

As an illustration of the orbit around a planet (ex. Earth), the much-used Newton's cannon model may prove useful (see image above). Imagine a cannon sitting on top of a (very) tall Mountain, about 40km, which fires a cannonball horizontally. The mountain needs to be very tall, so that the cannon will be above the Earth's atmosphere and we can ignore the effects of air friction on the cannon ball.

If the cannon fires its ball with a low initial velocity, the trajectory of the ball will curve downwards and hit the ground (A). As the firing velocity is increased, the cannonball will hit the ground further (B) and further (C) away from the cannon, because while the ball is still falling towards the ground, the ground is curving away from it (see first point, above). If the cannonball is fired with sufficient velocity, the ground will curve away from the ball at the same rate as the ball falls — it is now in orbit (D). The orbit may be circular like (D) or if the firing velocity is increased even more, the orbit may become more (E) and more elliptical. At a certain even faster velocity (called the escape velocity) the motion changes from an elliptical orbit to a parabola, and will go off indefinitely and never return. At faster velocities, the orbit shape will become a hyperbola.

IL 7 *** An interactive program for Newton's cannon thought experiment

Questions about satellite motion Newton could have answered.

Use IL 7, above, for answering these. (Note: You can obtain a qualitative and some quantitative understanding of satellite motion using this interactive program. Below we will discuss a quantitative approach, using Newton's own calculations.)

1. Find the velocity required to place a satellite into a circular orbit, not far above the surface of the earth.
2. Notice that velocities lower and higher than this velocity describe a motion that looks like an ellipse.
3. Find the escape velocity of an object from earth, the velocity necessary to escape the gravity of the earth.

IL 8 *** Air pressure and temperature

Preliminary Activity

In 1946 the US began a vigorous program of high-altitude rocket flights that carried scientific instruments into the stratosphere and ionosphere to measure temperature, density, magnetic field strength, data on the Northern Lights, etc... A decade later this work was given a boost by the International Geophysical Year (1957-58). The findings and new developments connected with these rocket flights served as the basis for later launchings of satellites into orbit, right after the Russians successfully launched their first satellite *Sputnik.* The Space Shuttle was developed twenty years later, as the ultimate vehicle to launch and retrieve small satellites on an on-going basis. The following is an example of a routine launch of an instrument package in that early period of space exploration.

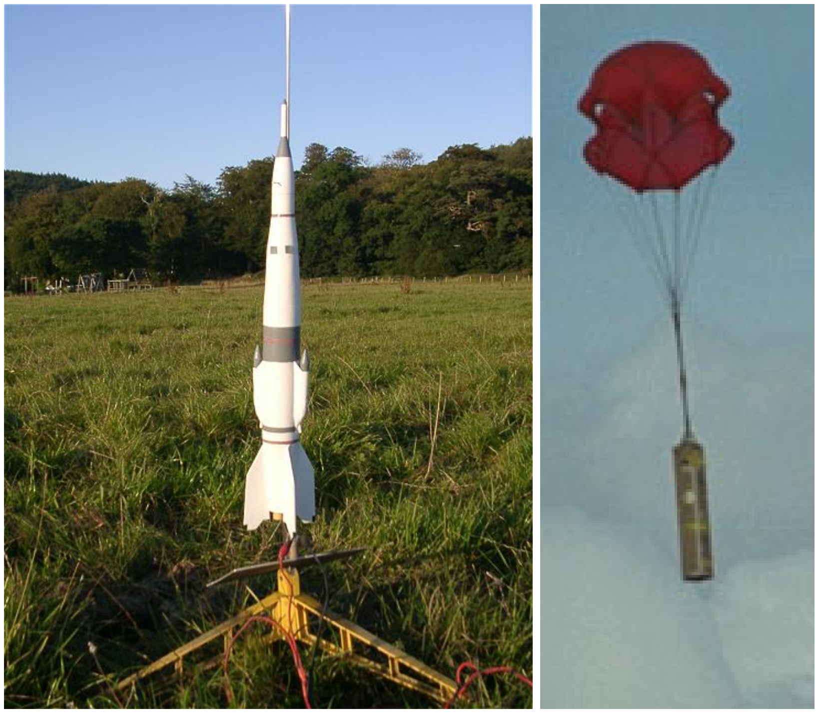


Fig. 4: A Rocket About To Launch a Scientific Instrument into the Stratosphere

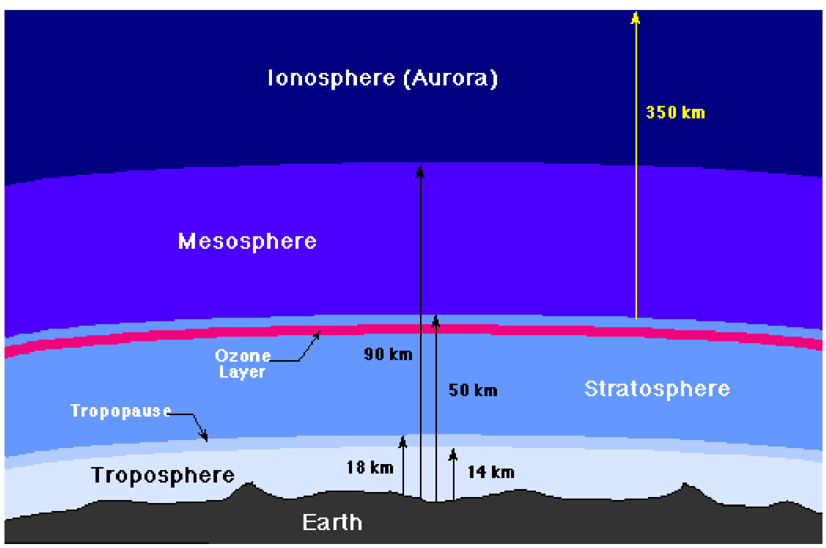


Fig. 5: The Atmosphere

Fig. 6: The Variation of the Density of the Atmosphere with Altitude

(Note: 1 m = 3.281 feet, and the density at see level is 1.23 kg/m^3)

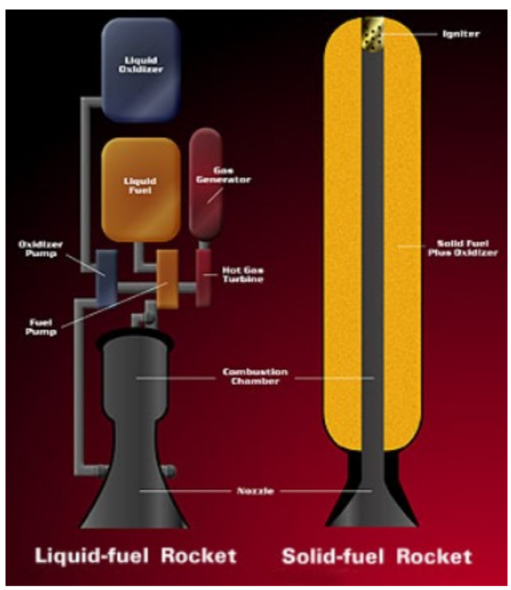


Fig. 7: Two Main Types of Rockets

For our first investigation we will discuss the trajectory of a small instrument package that is launched into the upper stratosphere (stratosphere) to obtain data on the ozone concentration bellow a height of 16 km. The instrument package, which has a mass of 10 kg, is launched vertically by a single stage rocket. The technical specs of the small rocket are given but most of these will be used later when we discuss this as an example of the physics of a simple rocket launch. See Fig. Our rocket uses a solid propellant which is a mixture of oxidizing and reducing materials that can coexist in the solid state at ordinary temperatures. When ignited, a propellant burns and generates hot gas expelled at a high velocity. Although gun powders are sometimes called propellants, the term solid propellant ordinarily refers to materials used to furnish energy for rocket propulsion.

Specifications for the Rocket and the Launch

Length……………………………….. 2.5 m.

Mass, fully loaded……………………. 200 kg

Mass of fuel…………………………... 130 kg

Mass of empty rocket………………… 60 kg

Mass of instrument package………….. 10 kg

Rate of mass ejection…………………. 2 kg/s

Ejection velocity of gases produced….. 1000 m/s

Area of parachute…………………….. 2.0 m^2

Drag coefficient for the tank………… 0.3

Drag coefficient for the parachute……. 1.0

Frontal area of the tank……………….. 0.10 m^2

The table of values, calculated, using the rocket equation, to be discussed later, is given below. These values can be calculated by using a calculator or, even better, applying the power of a data processor like Excel. The idealized velocity-time as well as the acceleration-time graphs for the motion of the instrument package is also shown below.

Table 1

Time (s)	v (m/s)	a (m/s^2)	m_i / m_f	m_f (kg)
1	0	0.1	1.01	198
5	0	0.72	1.05	190
10	6.4	1.3	1.11	180
15	15.5	2	1.18	170
20	27.1	2.7	1.25	160
25	42.7	3.5	1.33	150
30	62.7	4.5	1.43	140
35	87.8	5.6	1.54	130
40	119	6.9	1.67	120
45	157	8.38	1.82	110

50	**203**	**10.2**	**2**	**100**
55	**259**	**12.4**	**2.22**	**90**
60	**328**	**15.2**	**2.5**	**80**
65	**412**	**19.5**		**70**
70	**363**	**-9.8**		**10**
75	**314**	**-9.8**		**10**
80	**265**	**-9.8**		**10**
85	**216**	**-9.8**		**10**
90	**167**	**-9.8**		**10**
100	**69**	**-9.8**		**10**
107	**0**	**-9.8**		**10**

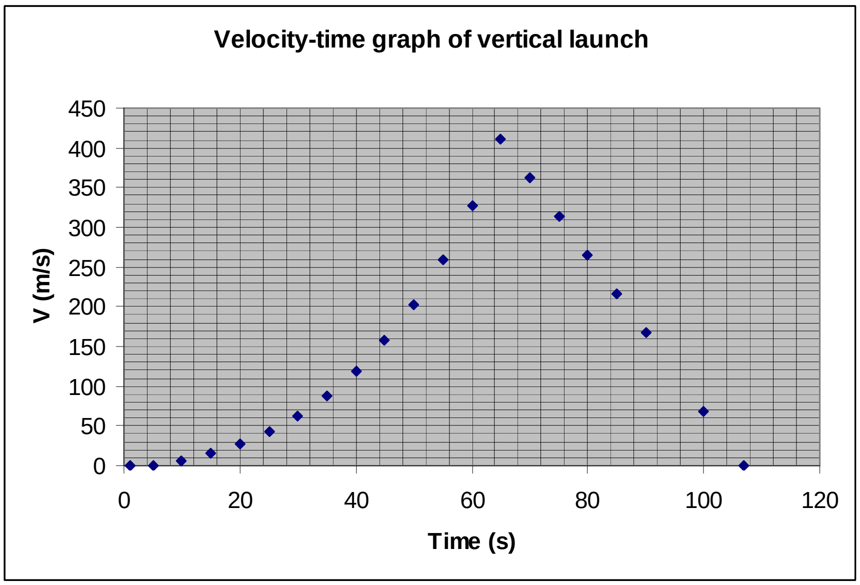


Fig. 8: The v-t Graph of the Launch

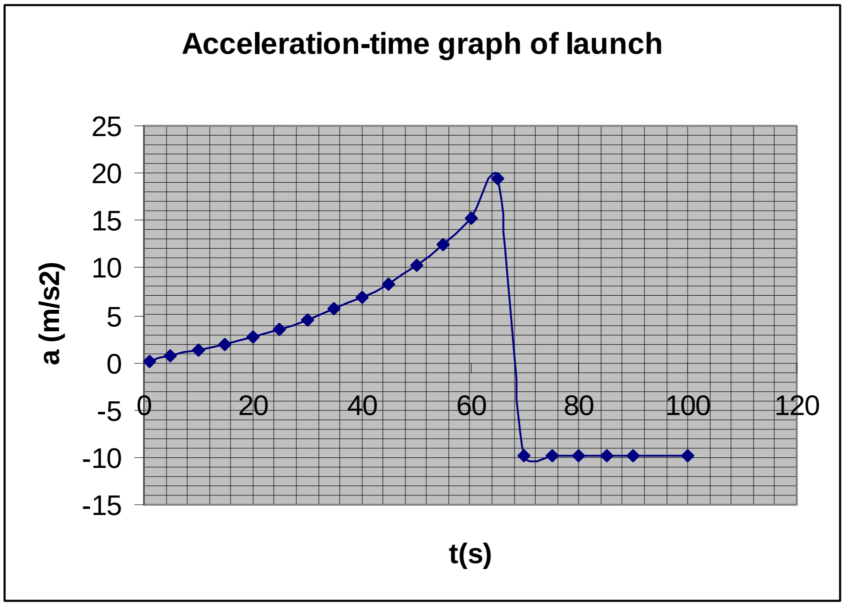


Fig. 9: The acceleration-time Graph for Launch

The Description of the Launch

According to the v-t graph, the instrument package reaches a height of about 8 km, when the fuel tank and the instrument package separate. Both the tank and the instrument package, of course, continue to rise to a height of almost 16 km. On falling back to earth, the parachute, attached to the package, opens when the velocity is about -40 m/s. The 2 square meter area of the parachute provides a drag force of about 100N, which will keep the velocity of the package at 40 m/s and then ensure the decrease of the velocity as the density of the air increases. The tank, however, will continue falling and reach a terminal velocity of about 56 m/s, just prior to collision with the surface of the earth. When the package makes contact with the ground (or water) in a "soft" landing with a speed of speed of about 9 m/s.

a. The Kinematics of the "flight"

We will first discuss the kinematics (or motion without considering the forces involved) of the flight. You must consult both the table of values and the graphs when solving these problems

We can make these calculations because the detailed velocity-time graph of the whole flight is given. You could either plot the graphs for the velocity and acceleration time, or enlarge the graphs given in the figures above.

<u>Questions and Problems</u>

1. At what time did the package reach maximum speed?
2. At what time did the package reach maximum altitude?
3. What was the altitude reached by the package at the end of the burn? At the time the

velocity of the package was zero?

4. What was the velocity of the rocket when it reached a height of 100m, 10000 m, 7000 m?
5. At what time did maximum acceleration occur? What was its magnitude?
6. Using the graphs show that the acceleration at time t can be found by determining the tangent of the v-t graph.
7. Estimate the total distance covered by the package as it was falling in still air. If there were a wind velocity of about 10 m/s in the Easterly direction.
8. Calculate the speed of an object falling from a height of 16000 m if there were negligible air resistance.
9. The parachute opens automatically when the speed of the package is about 40 m/s. The package then slows down to about 9 m/s before it makes contact with the ground. Estimate the time the package is in the air.
10. Sketch a v-t and a h-t and an a-t graph of the flight of the package. Place these graph appropriately on top of each other, d-t, v-t, and a-t grahs. Show how, by taking tangents going down starting with the height (distance) you can plot the graph of velocity –time and hen the graph of acceleration-time. Show that by going up from the acceleration-time graph and taking areas you can plot the graphs of the velocity-time and the distance-time graph. See Fig. 11 below).
11. Find the height to which the package rises at the end of burn-out.
12. Calculate the maximum height the package rises to before beginning descent.

Area Rule
(Going Up)

Slope Rule
(Going Down)

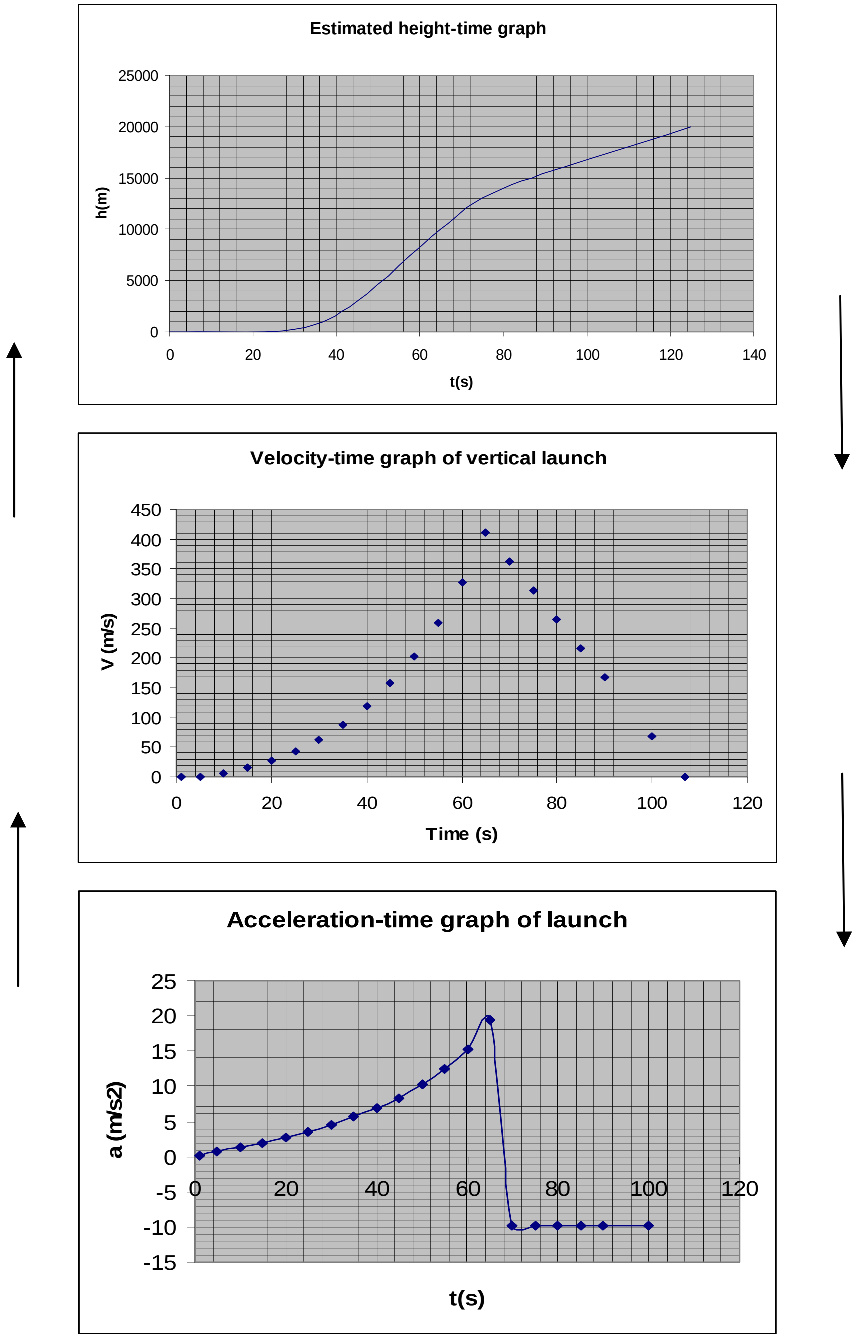


Fig. 10: h-t, v-t, and a-t Graphs of the Package Motion

Note: You can relate these three kinematic graphs by using the *area rule* for going up from the a-t to the v-t to h-t graph, and then by going down from the h-t, to the v-t, to the a-t graph using the *tangent rule.* See LCP 1 for a review of kinematics.

b. The Dynamics of the "Flight"

The dynamics of the motion requires understanding of the forces involved. In this simple case we don't have to worry about gravity and the forces produced by the rocket because we have a velocity-time and an a-t graph available. From the v-t graph we can easily determine the acceleration-time graph. Since force is directly proportional to acceleration, the force-time graph follows directly.

Questions and Problems

1. Between what times did the package experience the greatest force? Explain
2. Draw a corresponding acceleration-time (a-t) for the whole flight.
3. Draw a corresponding distance-time (d-t) graph for the whole flight.
4. Suggest what a v-t graph of a "real" launching would look like. Take into account air resistance and perhaps the change in the strength of the gravity.
5. In a realistic calculation of the trajectory must include drag experience d by the rocket.
6. Using the drag equation in LCP 3, show that the terminal velocity of the tank would be about 200 km/s and that of the package about 9 m/s.

Recent History of Rockets and Satellites

In 1928 Herman Potočnik (1898–1929) published his sole book, *Das Problem der Befahrung des Weltraums - der Raketen-Motor* (*The Problem of Space Travel - The Rocket Motor*), a plan for a breakthrough into space and a permanent human presence there. He conceived of a space station in detail and calculated its geostationary orbit. He described the use of orbiting spacecraft for detailed peaceful and military observation of the ground and described how the special conditions of space could be useful for scientific experiments. The book described geostationary satellites (first put forward by Tsiolkovsky) and discussed communication between them and the ground using radio, but fell short of the idea of using satellites for mass broadcasting and as telecommunications relays.

In a 1945 *Wireless World* article the English science fiction writer Arthur C. Clarke (b. 1917) described in detail the possible use of communications satellites for mass communications. Clarke examined the logistics of satellite launch, possible orbits and other aspects of the creation of a network of world-circling satellites, pointing to the benefits of high-speed global communications. He also suggested that three geostationary satellites would provide coverage over the entire planet.

The launching of a satellite varies in two ways. The first way of launching a satellite is to carry it into space on a shuttle like the Space Shuttle. The Space Shuttles carries some satellites into space. When the satellite is raised to the required height it is given a thrust into orbit.

However, there is another way of launching a satellite into space, this method of launching is done with a rocket. A satellite is put into a rocket and launched into space. This method of launching is to raise the projectile to the required height and then give it a side ways thrust which will push it into the right orbit. When a rocket is launched upward into space the rockets fuel is spent, the satellite separates from the rocket and the rocket falls from space and into the ocean. The satellite is left in space. The satellite requires minor adjustments; this is accomplished by built-in rockets called thrusters. When a satellite is up in orbit it stays in orbit for a long time.

IL 9 **** An excellent elementary discussion of launching a satellite

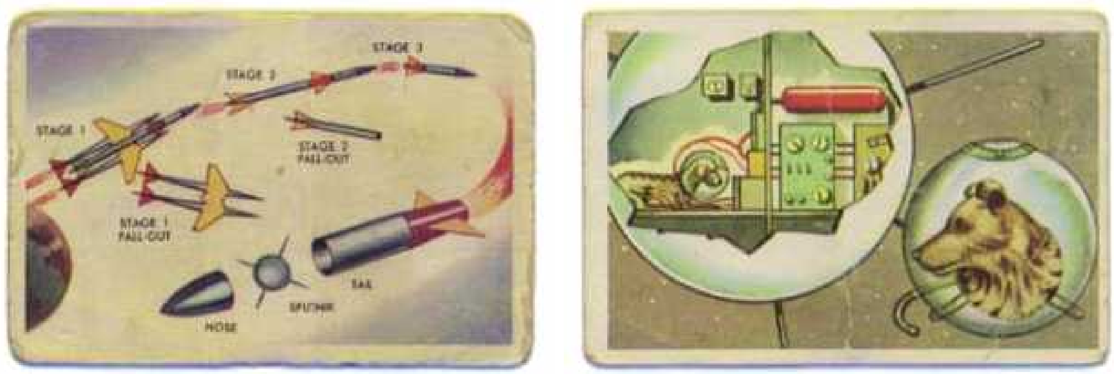


Fig. 11: Rocket Cards Showing Sputnik Launching and Inside Sputnik 2

IL 10 *** Interactive launch demo.

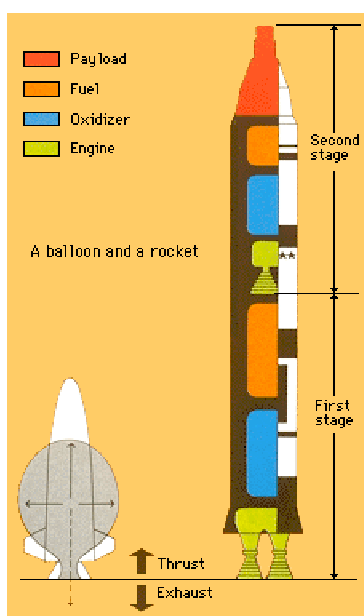


Fig.12: A Two-Stage Rocket Carries a Propellant and One or More Rocket Engines in Each Stage

The first stage launches the rocket. After burning its supply of propellant, the first stage falls away from the rest of the rocket. The second stage then ignites and carries the payload into earth orbit or even farther into space. A balloon and a rocket work in much the same way. Gas flowing from the nozzle creates unequal pressure that lifts the balloon or the rocket off the ground. Image credit: World Book diagram

IL 11 **** An excellent discussion of rockets

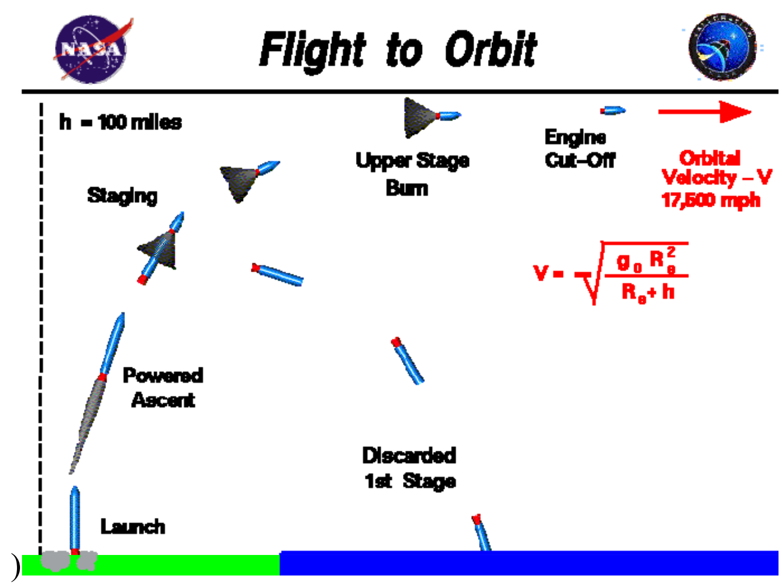


Fig. 13: A Two Stage Rocket Launch

IL 12 *** Ballistic flight equation, with an interactive applet

IL 13 *** Ballistic flight calculator

IL 14 *** An advanced discussion of a real ballistic flight. Flight calculator, including air resistance

IL 15 **** Source of Fig. 13 This is an excellent IL has a circular orbit calculator

After several minutes into the ascent, most launchers discard some of the weight of the rocket. This process is called staging and often includes the ignition of a second engine, or upper stage, of the launcher (See Fig. 13). The discarded first stage continues on a ballistic flight back to earth. The first stage may be retrieved, as with the Space Shuttle solid rocket engines, or it may be completely discarded, as was done on the Apollo moon rockets. The lighter, upper stage continues to accelerate under the power of its engine and to pitch over to the horizontal. At a carefully determined altitude and speed the upper stage engine is cut off and the stage and payload are in orbit. The exact speed needed to orbit the earth depends on the altitude, according to a formula that was already developed by Johannes Kepler in the early 1600's, a formula we will use again later. Using modern algebraic symbols Kepler's formula can be written as:

$$\mathbf{V = (g_0 * R_E^{\ 2} / (R_E + h))^{1/2}}$$

where **V** is the velocity for a circular orbit, $\mathbf{g_0}$ is the surface gravitational constant of the Earth (9.8 m/s^2), $\mathbf{R_E}$ is the mean Earth radius (6.4 x 10^6 km), and **h** is the height of the orbit in meters. If the rocket was launched from the Moon or Mars, the rocket would require a different orbital velocity because of the different planetary radius and gravitational constant. For a 100 mile (160km) high orbit around the Earth, the orbital velocity is 17,478 mph (27965 km/h, or 7.83 km/s).

IL 16 **** A Java interactive "Rocket Modeler" program

Questions and Problems

In preparation for our more detailed and quantitative analysis of a rocket launch, consider the following:

Study Fig. 13. and also Figs 14 and 17. Describe the motion of a rocket (from a vertical position) from the point of view if kinematics and dynamics. This is a large two-stage rocket that is able to place a satellite into a low earth orbit.

1. What should be the horizontal velocity of the final stage of the rocket when the satellite is paced into low-earth orbit at about 200 km?
2. What must be the minimum force acting on the rocket before lift-off is possible?
3. Draw a velocity-time graph of the motion, for the first stage of the ascent, where the speed reache is is typically about 2 km/s.
4. Draw an acceleration-time graph. Remember, the rocket is loosing mass as it is propelled and therefore the acceleration will change.
5. Draw a a force-time graph.(We are talking about the unbalanced force, acting on the rocket).
6. Describe the motion aftere the first stage has been discarded and the second stage of the rocket is activated. The rocket now deviates from the verical and begins moving at an an angle, eventually
7. Draw a picture of the second stage, showing the direction of the rocket slowly changing to the horizontal.
8. Separate the horizontal motion from the vertical motion and speculate on how a velocity-time graph would look for these motions.
9. Imagine a passenger like the dog in Sputnik. How would the weight of the dog change from the start to the time the dog entered the orbit?
10. Show that Kepler's equation $\mathbf{V = (g_0 * R_E^{\ 2} / (R_E + h))^{1/2}}$ can be obtained by combining the inverse square law of Newton and the definition of centripetal force. Then, how did Kepler arrive at this equation?

Part A: The Motion of the Rocket in Deep Space Where There Is No Gravitational Effect

We will solve this problem by using an approximation approach first and then compare the results to the full analytical solution. We are also imagining the rocket in deep space where the effect of gravity is negligible.

We begin by analysing the simple case of a rocket that ejects small, equal masses with an ejection velocity of v_e. First, we imagine the rocket ejecting 1/10 of its mass at a time until the last 1/10 of the mass remains. This is called the "payload". Then we will calculate (using MAPLE maths program) the case for ejecting 1/100 of the mass. Finally, we will show that in the limit of continuous ejection of mass we get the same answer as the one we would obtain using the method of the Calculus.

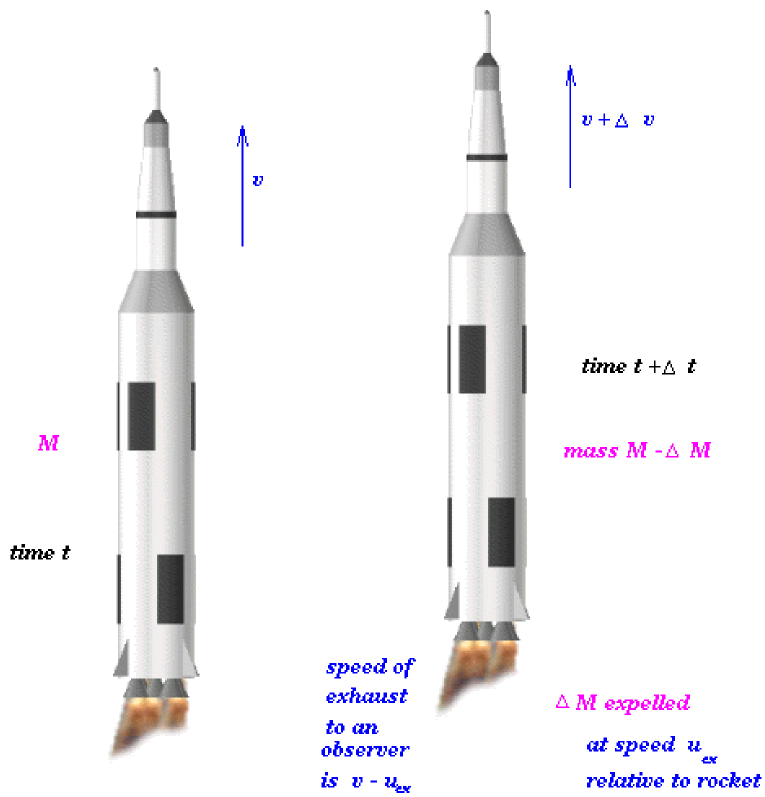
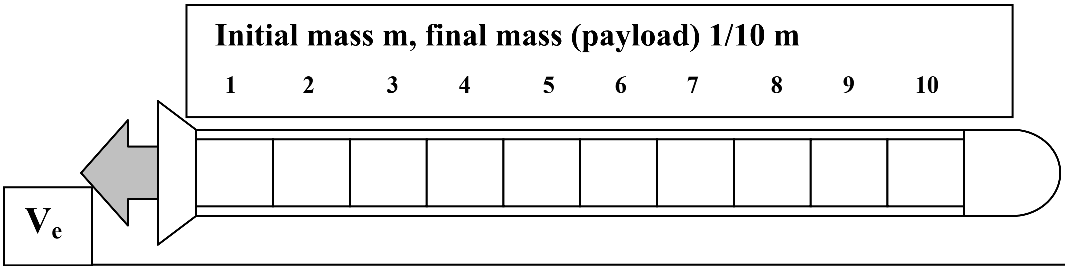


Fig. 14: Rocket in Deep Space (Negligible Gravity)

IL 17 *** Rocket launch calculator



Our rocket is divided into 10 sections, each with a **1/10** of the total mass of **m.** The rocket is propelled by ejecting each mass with a velocity of $\mathbf{v_e}$**,** relative to the rocket. The “payload” is the rounded part, of mass **Δm.** The rocket is in deep space.

Fig. 15: Simple Rocket Propulsion

An Approximation Approach

Our imaginary rocket, shown above, is unaffected by gravity and is propelled by ejecting each mass with a velocity of v_e, relative to the rocket. *In the rocket’s reference frame* at t = 0 the rocket is at rest. Using the conservation of linear momentum principle we can write:

$$\mathbf{P_B + P_A = 0, \text{ or } P_A = -P_B}$$

(Momentum before “explosion” = Momentum after “explosion”), or

$$\mathbf{v_e\, \Delta m = -\, \Delta v_{Rocket} / (m - n\Delta m),}$$

where n = 1,2,3,....

(The negative sign indicates that the rocket is moving in the opposite direction),

$$\mathbf{\Delta v_{Rocket} = -\, v_e\, \Delta\, m / (m - n\Delta m).}$$

The velocity of the rocket then will increase by

$$\mathbf{-\, v_e\, \Delta\, m / (m - \Delta m)}$$

for the first)m ejected. After the *second*)m is ejected the velocity will increase by the amount

$$\mathbf{\Delta m / (\, m - v_e\, 2\Delta m)}, \text{ etc.}$$

The final velocity of the rocket then, *as seen from the frame of reference that moved with the rocket when t = 0 is*:

$$\mathbf{v_f = -\, (\Delta\, v_1 + \Delta\, v_2 + \Delta\, v_3 + ...\, \Delta\, v_n)}, \text{ where}$$

Finally, we can write:

$$\mathbf{v_f = -\, \Sigma^{n=n}{}_{n=1}\, v_e\, \Delta m / (\, m - n\Delta m)}$$

For example, for the case of **n = 10,** or **Δm** = **m /10** we have:

$$\mathbf{v_f = v_e/10\ (\, 1/\, 9 + 1/8 + 1/7 + 1/6 + 1/5 + 1/4 + 1/3 + 1/2 + 1/1\,) = -\, 2.86\, v_e}$$

According to this simple model, the final velocity of the rocket (now reduced to a size of 1/10 of the original mass), called the "payload", will be 2.86 times the ejection velocity. The actual value, using the method of the calculus, however, is 2.30 v_e.

Using a maths program like MAPLE, we can easily show that if we let Δm be 1/100 m then we obtain a value very close to 2.3 v_e. Finally, if n = 4 the value is 2.30 v_e. This, of course, is the case for the rocket ejecting gas at a continuous rate of $\Delta m / \Delta t$, measured in kg/s.

The analytical equation (called "the rocket equation") we will see below is

$$\mathbf{v = v_e \ln (m_i/m_f)}$$

where **ln** is the *natural log* or the log to the base of e, or 2.71….and $\mathbf{m_i/m_f}$ is the ratio of initial mass to the final mass (the latter is often called the 'payload').

For our problem this ratio is 10, so that. Using the rocket equation we find that the final velocity for our simple case would $\mathbf{v = v_e \ln (10/ 1)}$ = **is 2.30** $\mathbf{v_e}$

<u>Finding the Instantaneous Acceleration of the Rocket</u>

You can easily show, using Newton's second law, that

$$\mathbf{T = R\, v_e = m\, a},$$

where **T** is the thrust produced by the gases being ejected, given in Newtons, **R** is the constant rate at which mass is ejected in (kg/s). We can now write:

$$\mathbf{R\, v_e = ma = (m_i - Rt)\, a},$$

where $\mathbf{v_e}$ is the escape velocity (relative to the rocket) of the gases. $\mathbf{m_i}$ the initial mass, **m** the mass at time **t** (**m** can be written as $\mathbf{m_i - Rt}$) and **a** is the acceleration at time t. Solving for a, then, we obtain:

$$\mathbf{a = R\, v_e / (m_i - Rt)}$$

The Analytical Approach

Using calculus (see, for example, *Fundamentals of Physics*, by Halliday and Resnick) it can be shown that $\mathbf{v = v_e \ln (m_i/m_f)}$. This is considered the most important equation of rocket propulsion, and should be memorized. We can derive this famous equation, using elementary calculus, this way:

As before, the principle of the conservation of linear momentum requires that

$$\mathbf{dm\, v_e = -mdv}$$

Rewrite and integrate between proper limits ;

$$\int v_0 dv = v_e \int_{m_i}^{m_f} \frac{dm}{m}$$

From which we obtain:

$$v_e \ln[m]_{m=m_i}^{m=m_f} \text{ or}$$

$$\mathbf{v = v_e \ln (m_i/m_f)}$$

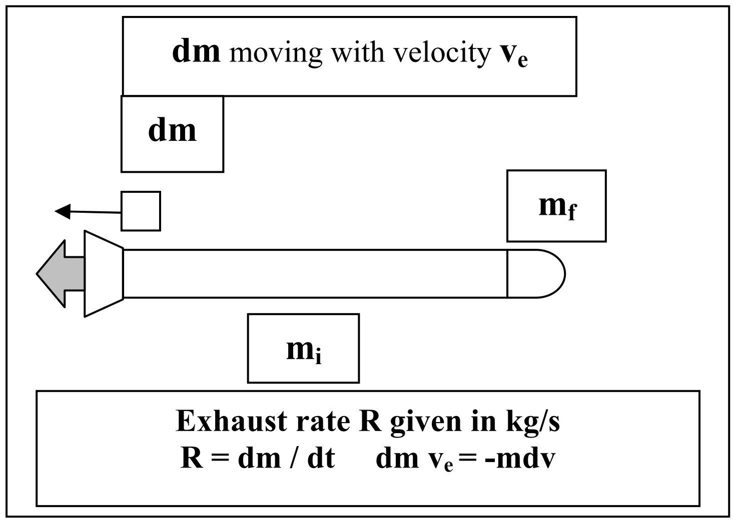


Fig. 16: The Physics of a Simple Rocket

For example, if the initial mass of a small rocket is 1000 kg and the final mass is 100 kg with a 90% of the total mass being fuel supply, the velocity of the "payload" will be v = v_e ln (10) = 2.30 v_e. A typical escape velocity for the gas is 2000 m/s. So the payload would be travelling at 4600 m/s, *relative to the original frame of reference of the rocket at t = 0.*

We have seen that the result we obtained above is a good approximation of this, especially if we take more than 10 Δm's. For example, if you take 100 Δm's, the final velocity would be very close to that calculated using the equation above.

IL 18 **** An excellent elementary discussion of rocket dynamics

Questions and Problems

Use the analytical formulas for all problems

1. A 10000kg rocket is launched in deep space from a space ship, directly ahead of the spaceship. The ejection velocity of the gases is 3000m/s at the rate of 5kg/s. The payload is to be 1000kg. (The mass of the spaceship is very much larger than the mass of the rocket).
   a. What is the initial acceleration of the rocket?
   b. What is the maximum acceleration of the rocket?
   c. What is the final velocity of the rocket relative to the spaceship?
   d. How far away is the rocket from the spaceship when the final velocity is reached?
   e. Speculate on how you could determine the time it took for the rocket to reach the highest velocity.

2. Is it possible to achieve velocities higher than the ejection velocity of the gases that propel the rocket? Explain. For what final mass will the velocity be equal to the ejection velocity?

Part B. The Motion of a Rocket, Launched On the Surface of the Earth, Where the Effect of Gravity Must Be Considered

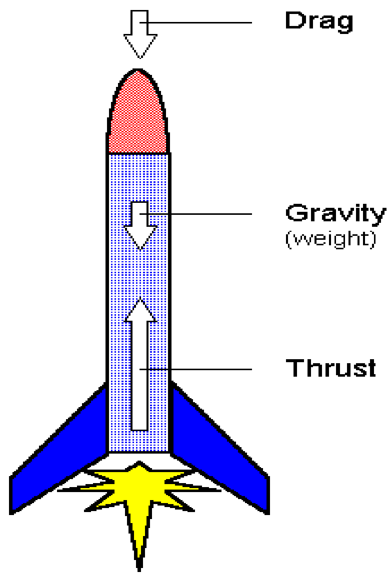


Fig. 17: A Rocket Rising Above the Surface of the Earth

When gravity acts on the rocket it is more difficult to find the final velocity in terms of the initial mass, final mass, and ejection velocity by an approximation approach like the one we used above. We cannot use the conservation of linear momentum any more. Why not?

An Approximation Approach

This time we will first find the acceleration of the rocket at time t:

Using Newton's second law we can write:

$$\mathbf{F = T - W}$$

Where **F** = unbalanced force acting on the rocket, **T** is the thrust, and **w** is the weight.

But $\mathbf{T = R\, v_e}$ and $\mathbf{w = mg}$ (we are assuming negligible air friction and a constant gravitational field). Next we find the mass of the rocket at time t:

$$\mathbf{m_t = m_i - Rt},$$

where **R** is the rate at which mass is ejected. Substituting and arranging terms we obtain:

$$\mathbf{a}_{(t)} = \frac{\mathbf{Rv_e - (m_i - Rt)g}}{\mathbf{(m_i - Rt)}}$$

This expression then gives us the acceleration as a function of time **t**.

To find the velocity of the rocket at time **t** we argue the following way:

We already showed that if the rocket were in gravity-free space the velocity would be:

$$\mathbf{v(t) = v_e \, \Sigma^{t=t}_{t=0} \, \Delta m \, / \, (m_i - Rt)}$$

But gravity affects the rocket and "pulls" it back by the amount of **gt** m/s. Therefore, the velocity at time **t** in a gravitational field will be:

$$\mathbf{v(t) = v_e \, \Sigma^{t=t}_{t=0} \, \Delta m \, / \, (m_i - Rt) - gt}$$

The Analytical Approach

Using calculus and integration, as we have already shown deriving the rocket equation as it would apply in deep space, we find, not surprisingly, that on the surface of the earth:

$$\mathbf{v = v_e \ln (m_i / m_f) - gt}$$

(Note: If we introduced a variable gravitational field and the effect of air friction you can imagine how complicated the problem would be!)

Revisiting Our First Rocket Launching

To prepare you for the dynamics of the actual Shuttle launch we will revisit our first rocket problem where we launched an instrument package into the lower stratosphere. Reread the section we discussed earlier without analyzing the dynamics of the trajectory. The specs for the launch are given again:

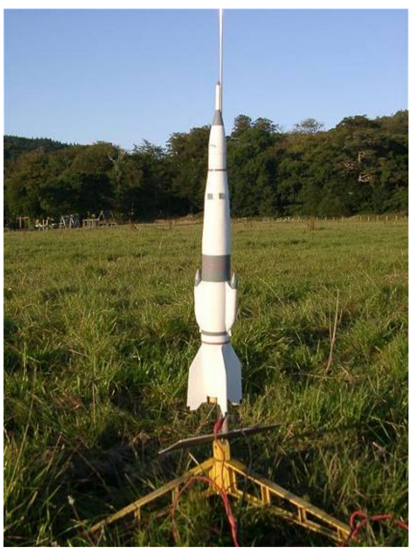


Fig. 18: A Model Rocket Ready for Launch

Specs for the rocket and the launch:

Length………………………………	2.5m
Mass, fully loaded…………………..	200 kg
Mass of fuel…………………………	130 kg
Mass of empty rocket……………….	60 kg
Mass of instrument package………….	10 kg
Rate of mass ejection…………………	2 kg/s
Ejection velocity of gases produced…	1000m/s
Area of parachute…………………….	2.0 m^2
Drag coefficient for the tank………….	0.3
Drag coefficient for the parachute…….	1.0
Frontal area of the tank……………….	0.10 m^2

Questions and Problems

Part A

1. Lift-off takes place when the trust is just a bit larger than the weight. Why?
2. What is the thrust on the rocket?
3. What would the minimum rate of ejection of the gases have to be if lift-off is to occur?
4. Find the initial acceleration of the rocket.
5. What is the maximum acceleration and when does it occur?
6. Find the final velocity of the tank and instrument package.

Part B

1. How long will it take for the fuel to burn?
2. Using the rocket equation, calculate a few values of the velocity and check your answer with the data given in the Table above.
3. Using the equation for thrust and acceleration in a gravitational field, calculate a few values of the acceleration and check your answer with the data given in the Table above.
4. The rocket will reach a height of about 16 km. What will be the gravity at this height? What percentage of the gravity on the surface of the earth?
5. Answer all questions above, if the same rocket were fired in deep space. Comment.
6. Using the v-t graph from the previous section calculate the height the rocket rises when burnout takes place (after 65 seconds).

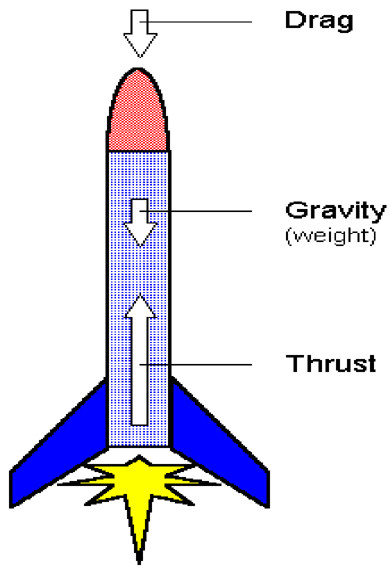


Fig. 19: A Rocket Rising Above the Surface of the Earth

Part C

In order to get a more realistic picture of the forces acting on the rocket, we must account for the drag force on the rocket as it moves through the atmosphere.

$$\mathbf{v = v_e \ln (m_i/m_f)}, \text{ and } \mathbf{T = R\, v_e}$$

From Fig. 19, it is clear that the total force on the rocket as it moves vertically (at time t) is given by

$$\mathbf{F_{net} = T - (D + W) = R\, v_e - (\tfrac{1}{2} A\, \rho\, v^2\, C_D + m_t\, g)}$$

where $\mathbf{m_t}$ is the mass at time t. This equation looks simple enough except that we have to guess the velocity for a given time t.

Here is an example:

Let's find the approximate net force on our rocket at 10, 25, 50, 70 seconds. From Table 1 we know that, when disregarding drag, the corresponding velocities are: 5, 38, 193, 400 m/s.

1. Calculate the drag force on the rocket at the following velocities. Estimate the height of the rocket at each velocity and use the appropriate density of the atmosphere, taken from the graph in Fig. 4 or Fig. 7.

   a. 10 m/s

   b. 50 m/s

   c. 100 m/s

   d. 400 m/s

Arthur C. Clark Proposes a Satellite Communication Network

We had mentioned earlier that in a 1945 Arthur C. Clarke described in detail the possible use of communications satellites for mass communications. Clarke examined the logistics of satellite launch, possible orbits and other aspects of the creation of a network of world-circling satellites, pointing to the benefits of high-speed global communications. He also suggested that three geostationary satellites would provide coverage over the entire planet.

We will now discuss the physics of placing three communication satellites in a geostationary orbit

A Geosynchronous (Geostationary) Orbit (GESO)

The following discussion is taken from the following two taken from

IL 19 *** Original prediction made by Arthur C. Clarke in 1945

IL 20 *** Original prediction made by Arthur C. Clarke in 1945

The concept was first proposed by the science fiction author Arthur C. Clarke in a paper in Wireless World in 1945, based on Herman Potočnik's previous work. Working prior to the advent of solid-state electronics, Clarke envisioned a trio of large, manned space stations arranged in a triangle around the planet. Modern satellites are numerous, unmanned, and often no larger than an automobile.

The first geosynchronous satellite was Syncom 2, launched on a Delta rocket B booster from Cape Canaveral 26 July 1963. It was used a few months later for the world's first satellite relayed telephone call, between U.S. President John F. Kennedy and Nigerian Prime minister Abubakar Tafawa Balewa.

The first geostationary communication satellite was Syncom 3, launched on August 19, 1964 with a Delta D launch vehicle from Cape Canaveral. The satellite, in orbit near the International Date Line, was used to telecast the 1964 Summer Olympics in Tokyo to the United States. It was the first television program to cross the Pacific Ocean.

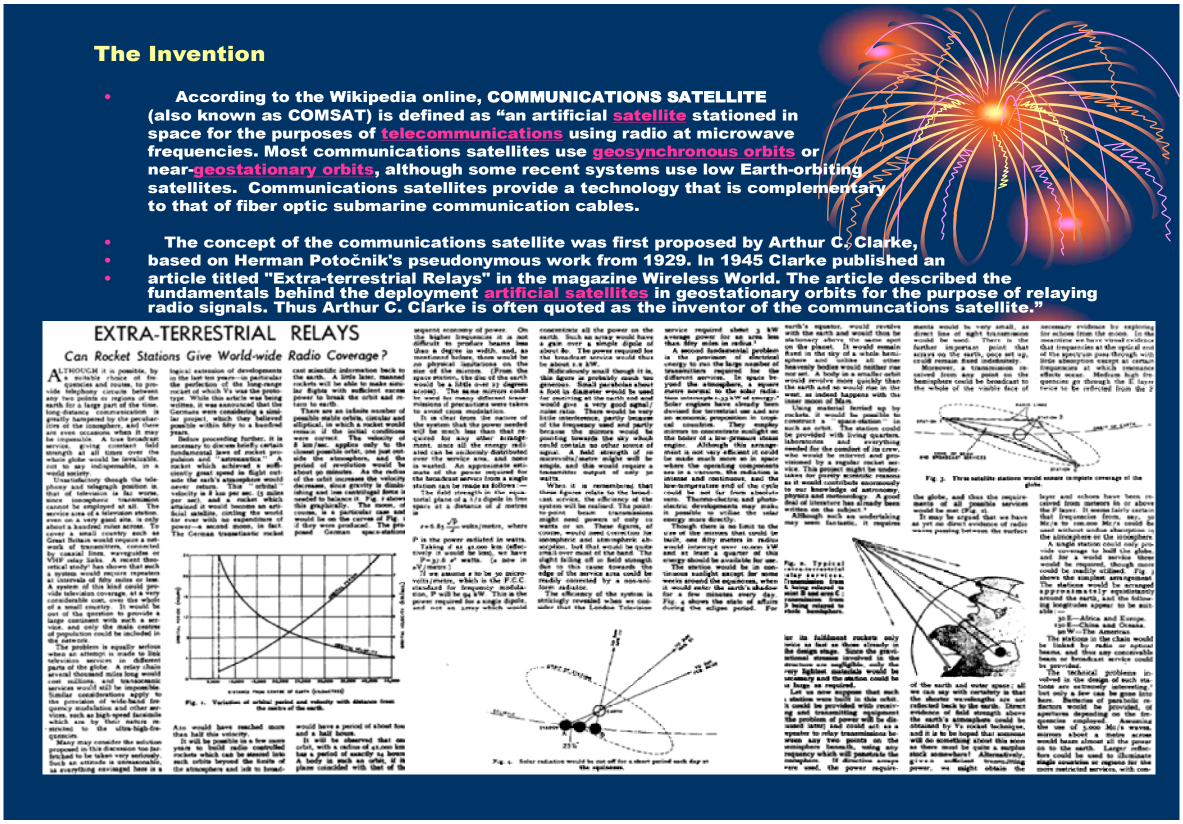


Fig. 20: The Original Article by Clarke, Published in 1945.

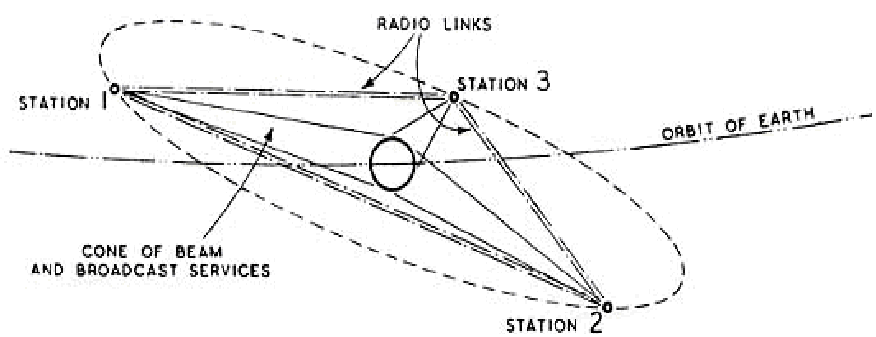


Fig. 21: A Sketch Showing Arthur C. Clarke's 1945 Proposal for World Communication, Using Three Geostationary Satellites

IL 21 **** Source of Fig. 22.

The geostationary orbit is a very special orbit, because the orbiting satellite stays in the same position *as seen from the surface of the Earth*. For this to happen we must find the orbit above the Earth and in the plane of the equator, that has a period of 24 hours. In this orbit there are many communication satellites. If it were not for these special satellites you could not point your satellite dish to one place and leave it there. Arthur C. Clark suggested that three communication satellites be placed at this distance from Earth, oriented at an angle of 120 degrees apart. In 1945, of course, this was science fiction.

IL 22 ** Good discussion of GCS

IL 23 ** Good discussion of GCS

The following discussion is taken from IL 22 and IL 23.

A **geo-stationary orbit** is an orbit of an Earth's satellite whose period of rotation is exactly equal to the period of rotation of Earth about its polar axis (which is 23 hours, 56 minutes and 4.1 seconds) and whose trajectory is aligned with the Earth's equator.

Any satellite in this orbit will appear as if it is always in the same place in the sky when observed from the same point on the Earth. This orbit is at a distance of approximately 35,900 km from the surface of the Earth. Communication satellites are usually placed into this orbit, with several satellites in the same orbit, distributed around to provide world wide coverage for relaying the telecommunication signals.

A geo-stationary orbit is also sometimes called: **stationary**, or **synchronous orbit**. One can also, launch a satellite into a synchronous orbit that is inclined to the Earth's equator. Once in this orbit, the satellite will trace a figure 8 once every 24 hours. The size and the shape of this figure will depend on the inclination angle.

Placing a Satellite in a Geostationary Orbit

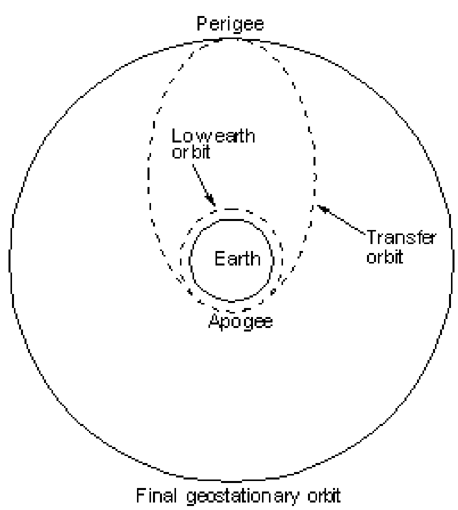


Fig. 22: Using a Hohmann Orbit to Place a Geostationary Satellite

In 1925, the German engineer-astronomer Walter Hohmann showed that the trajectory requiring the minimum energy to go to Mars would be the one shown in Fig. 22. We will discuss this special orbit in detail in LCP 7 (Journey to Mars).

The Hohmann trajectory between two circular (or near-circular) orbits is one of the most useful maneuvers available to satellite operators. It represents a convenient way of establishing a satellite in high altitude orbit, such as a geosynchronous orbit. For example, we could first position a satellite in LEO (low-Earth orbit), and then transfer to a higher circular orbit by means of an elliptical transfer orbit which is just tangent to both of the circular orbits. In addition, transfer orbits of this type can also be used to move from a lower solar orbit to a higher solar orbit; i.e., from the Earth's orbit to that of Mars, etc.

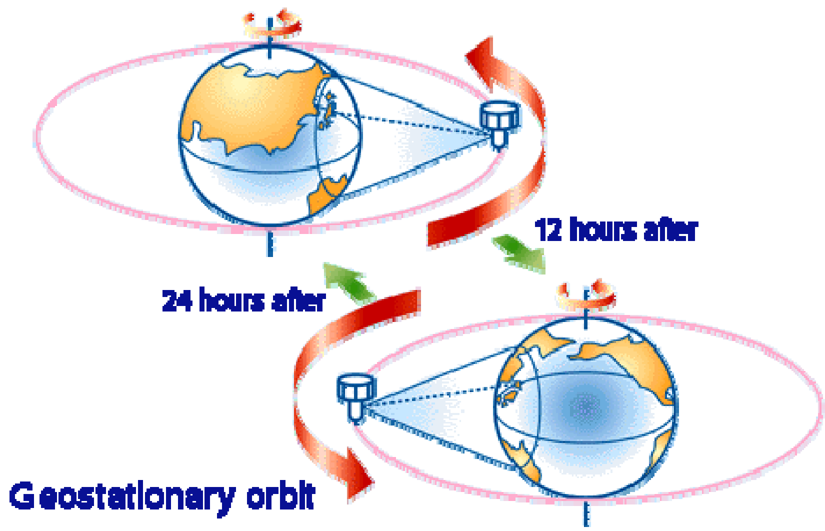


Fig. 23: The Position of a Geosynchronous Satellite at Twelve Hour Intervals

IL 24 *** Calculation of a geostationary orbit

Calculating the Position of A Geostationary Satellite

Refer to Fig. 24.

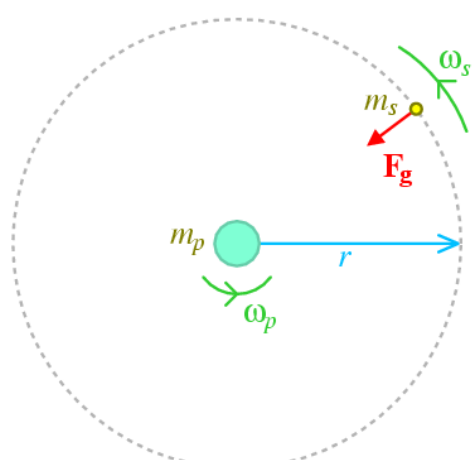


Fig. 24: The Physics of A Geostationary Orbit

Defining our terms:

m_s = mass of satellite (kg)

ω = angular velocity (radians per second), **ω** is defined as **v / r, v = ω r**

F_c = The centripetal force acting on the satellite

F_g = The force of gravity acting on the satellite

m_p = Mass of planet

m_s = Mass of satellite

Note that centripetal force can now be written as m **ω^2 r m** rather than **m v^2 / r**.

In order to keep a satellite of mass m_s in a circular orbit of radius r with a constant orbital angular velocity of ω_s, a centripetal force F_c needs to be supplied, where

$$F_c = m_s\,\omega_s^2\,r.$$

This will be provided by the force of gravitational attraction between the planet and the satellite,

$$F_g = G\,m_p\,m_s / r^2.$$

So, we have:

$$\begin{aligned} F_c &= F_g \\ m_s\omega_s^2 r &= \frac{Gm_pm_s}{r^2} \\ \omega_s^2 r &= \frac{Gm_p}{r^2} \\ r^3 &= \frac{Gm_p}{\omega_s^2} \\ r &= \sqrt[3]{\frac{Gm_p}{\omega_s^2}} \end{aligned}$$

The angular velocity of the satellite ω_s is related to its period of revolution T_s by

$$\omega_s = 2\,\pi / T_s$$

(remember that **v = 2 π r / T**)

For a geostationary orbit, we would like this period to be equal to the (sidereal) rotational period of the planet, ***T_p*.** We can therefore substitute ***$\omega_s = \omega_p = 2\,\pi / T_p$*** into the above equation to arrive at our final answer:

$$r = \sqrt[3]{\frac{Gm_pT_p^2}{4\pi^2}}$$

Substituting the values for

G = 6.67×10^{-11}

m_p = 5.975×10^{24} kg

T = 8.6164×10^{4} s

You should find that the radius (the distance from the center of the earth to the geostationary satellite is about 42000 km. Therefore the distance from the surface of the earth to the satellite is about 36000 km. This is about 6 times the radius of the earth!

<u>Testing Kepler's formula</u> (mentioned above):

We have seen earlier that Kepler's formula to find the velocity of a satellite in circular orbit, written in modern notation is:

$$\mathbf{v = (g_0 * R_E{}^2 / (R_E + h))^{½}}$$

where $\mathbf{g_0}$ is the gravity at the surface of the earth, and $\mathbf{g_h}$ is the gravity experienced at the height h above the earth. Using the inverse square law you can easily show that

$$\mathbf{g_h = g_0 * (R_E / (R_E + h))^2}$$

Since $\mathbf{F_g}$ (at R) $\mathbf{= m\ g_R}$ and $\mathbf{F_g = m\ v^2 / (R_E + h)}$ it follows directly that

$$\mathbf{v = (g_0 * R_E{}^2 / (R_E + h))^{½:}}$$

$$\mathbf{v = (g_0 * R_E)^{½}}$$

Finally, since $\mathbf{v = 2\ \pi\ r / T}$,

$$\mathbf{T = 2\ \pi\ r / v}$$

solving for T for the geostationary satellite we obtain:

$$\mathbf{T = 2\ \pi\ (R_E + h) / (g_0 * R_E{}^2 / (R_E + h))^{½}}, \text{ or}$$

$$\mathbf{T = 24\ hours}$$

and solving for T for the hypothetical satellite orbiting a perfect earth just above the surface, we obtain:

$$\mathbf{T = 2\ \pi\ R_E / (g_0 * R_E)^{½}}, \text{ or}$$

$$\mathbf{T = 84.\ 5\ minutes}$$

You will remember that in LCP 1 we found that a pendulum of the length of the earth radius would have a period of 84.5 minutes at the surface of the earth.

You can now show that when h = 0, that is the velocity of a satellite near the surface of the earth (imagine a perfect sphere and no atmosphere) would be just under 8 km/s.

<u>More Problems and Questions</u>

1. What is the gravity at the height of 36,000 km in m/s^2?
2. The moon is about 3.84×10^8 m from the center of the earth. Where between the earth and the moon would the gravitational effects cancel? Why is this considered an important location for future space flights?
3. Calculate the orbital velocity of the geostationary satellite. Comment about the change in velocity as a function of height.
4. Calculate the orbital velocity of a hypothetical satellite just above the earth. Show that the

satellite has a velocity of about 7.9 km/s

5. Consider the plan of placing three communication satellites, 120 degrees apart (See Fig. 21). Specifically, show that for a height h above the earth the orbiting velocity for a circular orbit is given by $\mathbf{V = V_0 (R_E / (R + h))^{1/2}}$.
6. Using the equation in #4, show that the velocity orbital velocity of the geostationary satellite can also be calculated using this equation.
7. Show that if the height of the satellite is equal to the radius of the earth, the velocity of the satellite would be given by $\mathbf{V = V_0 / 2^{1/2}}$
8. Sketch a graph of the velocity as a function of height. Comment.

THE FLIGHT OF THE SPACE SHUTTLE

IL 25 **** A complete history of the space shuttle

IL 26 **** A comprehensive history of the Shuttle

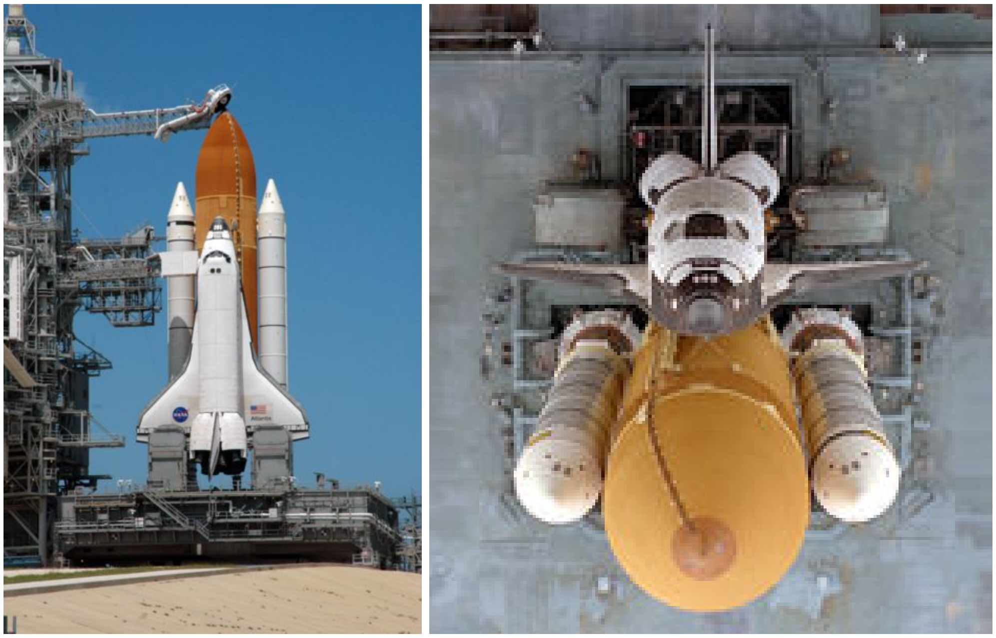


Fig. 25: Space Shuttle Atlantis on the Launch Pad

A front and an overhead view of *Atlantis* as it sits atop the Mobile Launcher Platform (MLP) can be seen in Fig. 25. The Shuttle "Stack" consists of Orbiter (on top), External Tank (at center), and Solid Rocket Boosters (to the right and left of External Tank). Two Tail Service Masts (TSMs) to either side of the Orbiter's tail provide umbilical connections for propellant loading and electrical power.

IL 27 *** Comprehensive and detailed history of the Space Shuttle program

Introduction

The first space shuttle flight took place in 1978. This was a culmination of a major national effort in the US that involved over 50,000 skilled workers in 47 states and almost 200 contracts, each worth more than a million dollars each ((1978 dollars). The four-part vehicle package consists of

1. An orbital space craft that resembles a stubby jetliner, and is designed to land like an airliner.
2. An expendable external fuel tank that supplies the propellants to the Orbiter's three liquid oxygen and liquid hydrogen engines.
3. Two reusable solid rocket boosters that provide the main initial ascent thrust.

The Space Shuttle was designed to make space easier to reach. It has been described as a "space truck" that carries cargo into space. The Space Shuttle is able to take large payloads into space and return them to be reused. Experiments can be performed in microgravity over long time periods. It also participated in the building of the new space station that orbits the earth.

The lifetime of the Shuttle is from 10 to 15 years. Placing and maintaining the Hubble space telescope, launching satellites and repairing those that malfunction and assisting the construction of the space station are only some of the many uses of the Shuttle. The Shuttle has been the place for hundreds of experiments in microgravity for physics, biology, chemistry and human physiology.

Conversions

1 nautical mile = 1.12 miles

1 mile = 5280 feet

1 mile = 1600 m

1 foot = 12 inches

1 inch = 2.54cm

1 m = 3.28 ft

1 km/h = 0.28 m/s

1 mile/h = 1.6 kmh

A Brief Glossary of Space Shuttle Terms

Abort .... the end of a mission short of its objective. An abort is caused by some malfunction or emergency.

Apogee .... the highest point of an earth orbit

g-force .... Forces produced on the body by acceleration, measured in terms of earth gravity

Geostationary orbit...... An orbit 22 300 miles (35 900 km) from earth, where the orbital period is 24 hours.

Geosynchronous orbit.. See "geostationary orbit"

Knot.............................. One nautical mile per hour, or 1.12 miles per hour

Microgravity................. The term used to describe the apparent weightlessness in orbit.

Mach .... The term used to describe the speed of objects relative to the speed of sound. (Named after Ernst Mach, a prominent German physicist of the 19th century)

Perigee........................... The point of closest approach of a satellite in an elliptical orbit

Thrust............................ The force created by a rocket engine

Note: These terms require a little discussion.

Terms .... (Acronyms)

ET-SEP.......................... External Tank Separation

FC .... Flight Computer

MECO .... Main Engine Cut-off

SRB................................ Solid Rocket Booster

SSME............................. Space Shuttle Main Engine

OMS .... Orbit Manoeuvring System

NASA's Space Shuttle includes a reusable manned spacecraft capable of delivering up to 25,000 kg of cargo into low Earth orbit. The four primary elements are the Orbiter (Rockwell), two Solid Rocket Boosters (Thiokol), External Tank (Lockheed) and three Space Shuttle Main Engines (Rocketdyne). A crew of up to eight (minimum two) is accommodated for up to 16 days in a shirt-sleeve environment. Cross range maneuvering capability during unpowered descent to a runway landing is 2,035 km; acceleration loads do not exceed 3 g during ascent or reentry.

The Launching of the Space Shuttle

In operation, the Shuttle is launched vertically with all main engines firing. At an altitude of about 45 km, after some 2 minutes, the boosters separate for recovery and refurbishment. The Orbiter continues under SSME power until about 8 min 50 s after launch, when the external tank separates for destructive reentry. Earlier missions required two OMS burns to attain operational orbit but a direct ascent technique is now employed, omitting the OMS-1 burn and relying on the OMS-2 burn at apogee about 45 min after launch.

IL 28 **** Nasa's Space Shuttle basics

The following is taken from the IL 28 above. The student should look at this excellent website and study it in detail.

IL 29 **** An excellent and detailed description of the Shuttle trajectory

IL 30 *** A complete description of a Shuttle trajectory

Fig. 26: Typical Space Shuttle Mission Profile

Orbiter: Discovery
Mission: STS-114
Launch: July 26 @ 10:39 a.m. EDT (1439 GMT)
Site: Pad 39B, Kennedy Space Center, Florida
Landing: Aug. 9 @ 8:11 a.m. EDT (1211 GMT)
Site: Shuttle Landing Facility, KSC)

IL 31 **** Timetable of a launch

IL 32 *** A good description of the trajectory of the Shuttle

Fig. 27: The Shuttle, Seconds after Liftoff

This image shows the space shuttle some distance off the ground and accelerating quickly through the atmosphere. Notice that the three main engines of the Orbiter are not producing smoke. Notice too how the external tank, solid rocket boosters and the Orbiter are attached to one another. The blue triangles just below the main engines are called “blue mach diamonds”. Their presence and distance from the main engines is an indication of a good liftoff.

Fig. 28 Launching of the Shuttle

First Stage Ascent

(Consult the velocity and acceleration time graphs while reading the description of the launch.) It takes five seconds for the shuttle to clear the 85 metre (247 feet) tower and its 30 m

(100 foot) lightning rod. By the end of the eighth second, the shuttle has traveled only twice it own length in distance has already accelerated to 161 kilometers per hour (100 mph.) During this short time, the Orbiter’s three main engines and two solid rocket boosters have consumed more than 680,000 kilograms (1.5 million pounds) of fuel.

About 20 seconds into the flight, the shuttle makes an unusual move. It rolls! The whole shuttle, also called the stack, turns so the Orbiter lies under the external fuel tank and the solid rocket boosters (see Fig. 25). This roll is important for a number of reasons. First, it reduces the stress on the Orbiter’s delicate wings and tail created by the near mach one speed of the shuttle at this point into the flight. Secondly, it makes it easier for the computer to control the shuttle during the remainder of the ascent. Thirdly, it enables the astronauts to see the horizon, giving them a reference point should the mission have to be aborted and the shuttle forced to land.

Shortly after clearing the tower the Shuttle begins a roll and pitch program to set its orbital inclination and so that the vehicle is below the external tank and SRBs, with wings level. The vehicle climbs in a progressively flattening arc, accelerating as the weight of the SRBs and main tank decrease. To achieve low orbit requires much more horizontal than vertical acceleration. This is not visually obvious since the vehicle rises vertically and is out of sight for most of the horizontal acceleration. The near circular Orbital velocity at the 380 km (236 miles) altitude of the International Space Station is 7.68 km per second (27,648 km/h, 17,180 mph), roughly equivalent to Mach 23 at sea level. For missions towards the International Space Station, the shuttle must reach an azimuth of 51.6 degrees inclination to rendezvous with the station.

It should also be mentioned that about 20 seconds into the flight, the shuttle has completed its roll and is accelerating through the atmosphere at about a 78 º angle. Stress on the Shuttle caused by its speed through the atmosphere is further relieved by powering back the main engines to about 70%. By 45 seconds into the flight, the shuttle breaks the sound barrier. A minute into the flight, the pressure on the Orbiter decreases and so the shuttle engines are returned to full power. At this point, the shuttle is traveling at an incredible 1,609 kilometers per hour (1,000 mph) or about Mach 1.5. By the end of the next minute, it will triple this speed!

Two minutes into the ascent, the shuttle is about 45 kilometres (28 miles) above the earth’s surface and is traveling nearly 5000 kilometers per hour (3,000 mph). The shuttle’s solid rocket boosters, having used their fuel, are commanded by the shuttle’s onboard computer to separate from the external fuel tank. Still propelled by their momentum, the spent solid rocket boosters will continue upward, but away from the shuttle, for another 11 kilometres (7 miles) before falling back to earth. Parachutes ejected from the nose cone of the rockets will slow their descent into the ocean some 225 kilometres (140 miles) off the Florida coast. Like the Orbiter, the solid rocket boosters are reusable. They will be retrieved, returned to the Kennedy Space Center and shipped to the manufacturer for refurbishing and refueling for a later shuttle mission. The jettison of the booster rockets marks the end of the first ascent stage and the beginning of the second. The solid fuel used by the boosters is actually powdered aluminum -- a form of the same metal you find in foil wraps in your kitchen -- with oxygen provided by a chemical called ammonium perchlorate, a powerful oxidizing agent.

After 126 seconds after launch, explosive bolts release the SRBs and small separation rockets push them laterally away from the vehicle. The Shuttle then begins accelerating to orbit speed drive by the Space Shuttle main engines. The vehicle at that point in the flight has a thrust to weight ratio of less than one — the main engines actually have insufficient thrust to exceed the force of gravity, and the vertical speed given to it by the SRBs temporarily decreases. However, as the burn continues, the mass of the propellant decreases and the thrust-to-weight ratio exceeds 1 again and the ever-lighter vehicle then continues to accelerate toward orbit (see velocity graph of the launch).

Second Stage Ascent

The second stage of ascent lasts about six and a half minutes. With the solid rocket boosters jettisoned, the shuttle is now powered solely by its three main engines. For the next six minutes, the shuttle will gain more altitude above the earth and more importantly, the speed of 28, 947 kph (8.10 km/s, or 17,500 mph) required to achieve orbit around the earth.

When the shuttle reaches a height of 100 kilometres (60 miles) above the earth's surface, its flight path levels out. It will fly more horizontally to the Earth and gain speed as it continues.

The three space shuttle main engines, attached to the rear of the shuttle Orbiter, continue to fire until about 8.5 minutes after liftoff, burning a half-million gallons of liquid propellant taken from the large, orange external fuel tank as the shuttle accelerates. The main engines burn liquid hydrogen — the second coldest liquid on Earth, at minus 252.7 degrees Celsius (minus 423 degrees Fahrenheit) — and liquid oxygen. Since the hydrogen and oxygen can reach a temperature as high as 3,315.6 degrees Celsius (6,000 degrees Fahrenheit) as they burn — higher than the boiling point of iron — the engines operate at greater temperature extremes than any other piece of machinery ever built.

The engines' exhaust is primarily water vapor as the hydrogen and oxygen combine. As they push the shuttle toward orbit, the engines consume the liquid fuel at a rate that would drain an average family swimming pool every 25 seconds and they generate over 37 million horsepower.

Finally, in the last tens of seconds of the main engine burn, the mass of the vehicle is low enough that the engines must be throttled back to limit vehicle acceleration to 3 g, largely for astronaut comfort (see velocity-time graph).

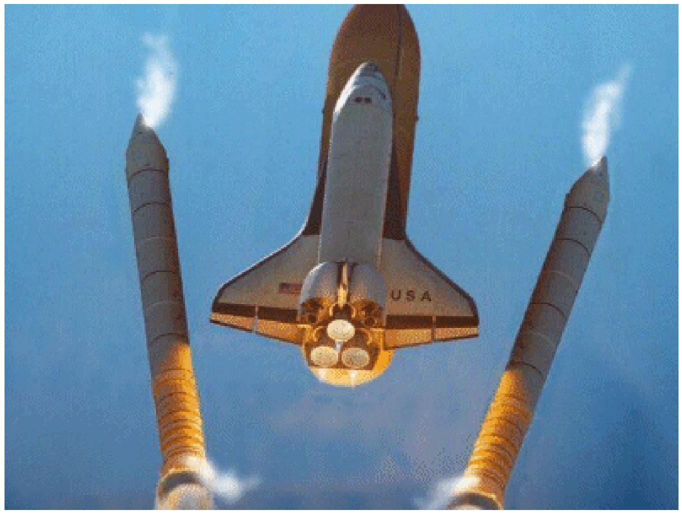


Fig. 29: The Boosters Are Discarded At About 2 Minutes into the Flight

Eight and a half minutes after launch, with the shuttle traveling just under 8 kilometers (5 miles) a second, the engines shut down as they use the last of their fuel. A few seconds after the engines stop, the external fuel tank is jettisoned from the shuttle. The only part of the shuttle that is not reused, the tank re-enters the atmosphere and burns up over the Pacific Ocean. The shuttle Orbiter, the only space shuttle component that will circle the Earth, weighs only about 117,934 kilograms (260,000 pounds). The shuttle has consumed more than 1.59 million kilograms (3.5 million pounds) of fuel during its first 8 ½ minutes of flight

Fig. 30: The Separation of the ET (External Tank)
After 8 ½ Minutes into the Ascent Orbit

Eight minutes into the flight, the shuttle reaches near orbital altitude. The main engines are commanded by the onboard computer to reduce power to ensure forces on the shuttle and its astronauts do not exceed 3 g's. Within thirty seconds the main engines are shut down completely. For the next eleven seconds, the shuttle and the external tank coast through space. At nine minutes, the command to jettison the nearly empty external tank is given by the computer. To avoid bumping into the freed tank as it begins to tumble towards earth, the shuttle's maneuvering rockets move it out of harms way. Gradually, because of the gravitational pull of the earth, the tank re-enters the earth's atmosphere. What doesn't burn up during reentry, crashes into the Indian Ocean or Pacific Ocean, depending on the trajectory of the shuttle's flight from lift-off.

After the main engines shut down, the shuttle is in an egg-shaped orbit (ellipse) that, if nothing changed, would cause it to re-enter the atmosphere above the Pacific Ocean, the same as what happens to the external fuel tank. But, about 35 minutes after the main engines have shut down, usually when the shuttle has reached the highest point of the egg-shaped orbit, the two orbital maneuvering system engines, located on the left and right side of the shuttle's tail, are fired for about three minutes. The orbital maneuvering system engines use two propellants that automatically burn whenever they contact one another, and the three-minute firing circularizes the shuttle's orbit at a safe altitude, one that will keep it above the atmosphere.

Fig. 31: The Orbiter in Orbit around the Earth

Fig. 32: Microgravity in the Shuttle

The shuttle is the only spacecraft ever built that can retrieve large satellites from orbit and bring them back to Earth. Using the Canadian-built robotic arm, called the Remote Manipulator System, mounted on the left-hand edge of the cargo bay, shuttle crews can move large objects into or out of the payload bay. The arm also can maneuver spacewalking astronauts into positions for satellite repairs and maintenance, as have been performed on the Hubble Space Telescope, or space construction, as is being conducted for the International Space Station.

The largest shuttle crew ever flown numbered eight people, but the average crew ranges from five to seven people. Crew members include pilot astronauts, called the commander and pilot who fly the shuttle, and mission specialist astronauts who are scientists and engineers trained to conduct the experiments onboard or perform specific tasks in orbit. Occasionally, the crew also may include payload specialist astronauts in charge of the operations of a specific cargo. The shuttle can launch as much as 28,803 kilograms (63,500 pounds) of cargo into orbit. It has remained in orbit for as long as 17 days before returning to Earth.

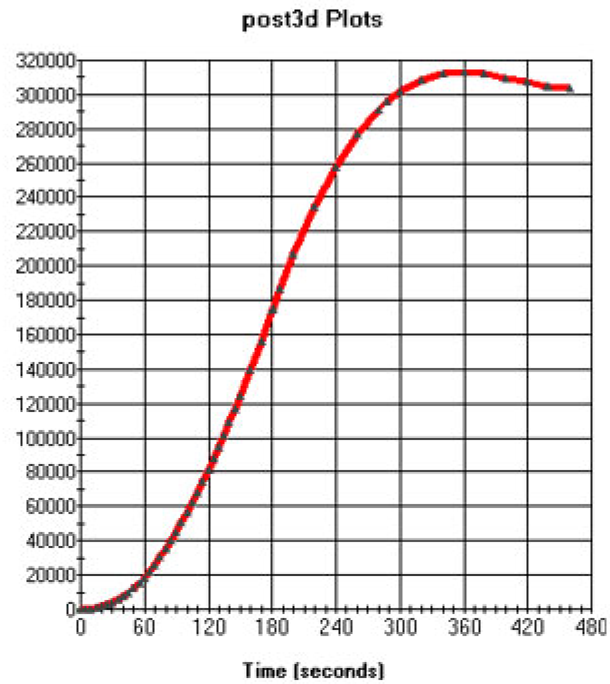


Fig. 33: Height-Time Plot (By NASA) Of the First 8 Minutes. The Height Is In Feet.

Summary of the Shuttle flight Sequence

Pre-launch.................................... part of the flight sequence of the space shuttle before launch.

Launching.................................... start of the ascent of the shuttle.

External tank separates.............. the external tank detaches from the Orbiter.

Solid rocket boosters separate ... separation of the external solid-rocket boosters.

Orbit around the earth............... repeated circling of the planet

Orbital operation........................ work to be done while the Orbiter is in orbit.

End of mission............................. all experiments have been completed.

Leaving earth's orbit.................. the Orbiter leaves orbit to return to earth.

Landing.. the Orbiter sets down on earth.

Rockets parachute into sea......... the rocket, emptied of its propellants, falls in ocean before being recovered.

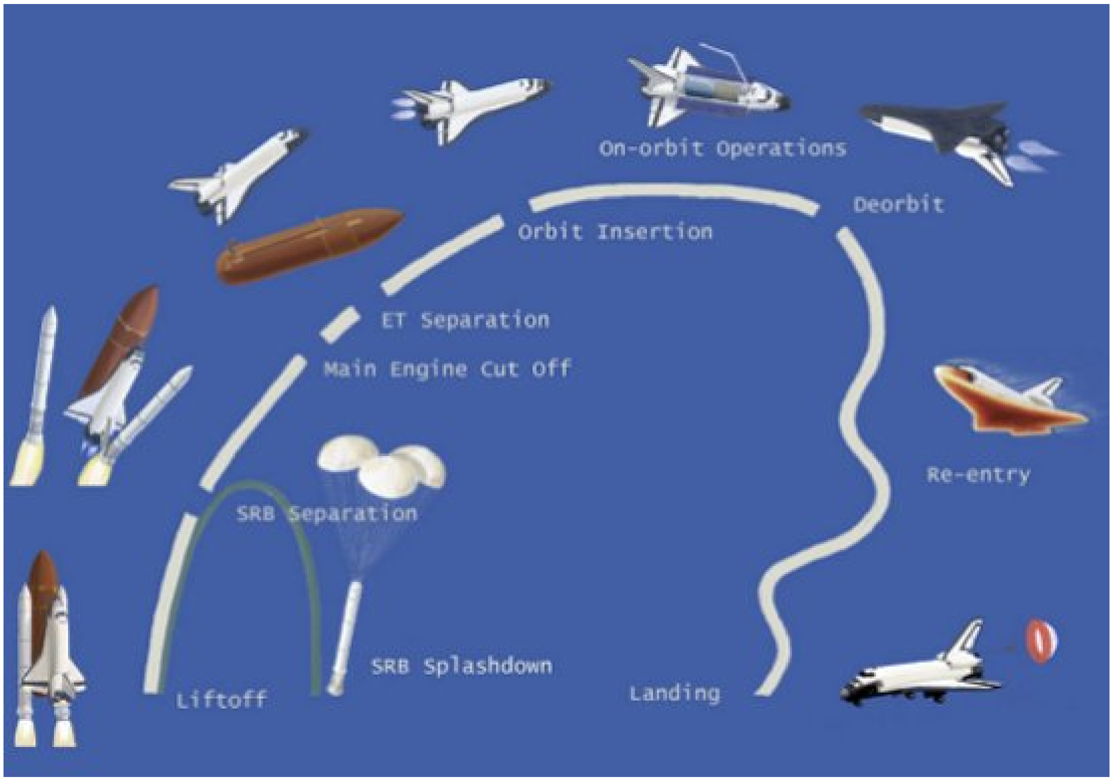


Fig. 34: The Shuttle Trajectory

IL 33 *** Videos of flight of the Shuttle

IL 34 *** Video of the Aug. 8 2007 launch of Endeavour

Using Elementary Physics to Describe the Kinematics and Dynamics of the Space Shuttle Launch

Table 2, below, shows the partial data taken from an actual Space Shuttle launch, taken from the IL 35 below, Students should refer to this table for more detail. The table only shows a few important data points that can be used for graphing. Notice that the data in the table below are in metric (SI) units of m/s, kg, and km. Unfortunately, NASA still uses the British system. This practice has led to a lot of major and expensive problems (accidents).

Note that the rest velocity of the Shuttle is not zero, but 410 m/s, and is called the inertial velocity of the Shuttle. So that if you want to find the velocity of the Shuttle relative to the earth you always have to subtract 410 m/s for this launch. Can you guess why this is done?

IL 35 **** Trajectory data for the Shuttle

The launch information to be considered below is as follows:

Orbiter: Discovery

Mission: STS-114

Launch: July 26 @ 10:39 a.m. EDT (1439 GMT)

Site: Pad 39B, Kennedy Space Center, Florida

Landing: Aug. 9 @ 8:11 a.m. EDT (1211 GMT)

Site: Shuttle Landing Facility, KSC

Table 2: A Partial Description of the Actual Shuttle Flight, Described Above

Time (Min. sec.)	**Velocity (inertial) Actual velocity** (km/s)	**Altitude** (km)	**Range** (km)	**Acceleration** (m/s^2)
00:00	0.410 0	0	0	4.0 at .25 min
00:30	0.531 0.121	2.71	0.96	6.4 .75 min
1:00	0.724 0.314	10.8	6.2	14.1 1.25 min
1:30	1.147 0.737	25.0	19.2	16.2
2:05	1.633 1.223	47.1	51.4	4.89
2:06 **SRB SEPARATION**	**SRB SEPARATION**			
2:30	1.779 1.369	61.8	82.0	7.4
3:00	2.000 1.590	76.6	125.3	8.7
3:30	2.262 1.852	88.8	179	10.1
4:00	2.565 2.155	97.6	237	11.9
Time (Min. sec.)	**Velocity (inertial) Actual velocity** (km/s)	**Altitude** (km)	**Range** (km)	**Acceleration** (m/s^2)
4:05---4:20 4:30	**negative Return** 2.923	104	309	12.9
4;55 **Partial** 5:00	**Orbit Possible** 3.311 2.515	107	390	14.6
5:30	3.751 3.341 (Mach 10)	109	483	16.7
6:00	4.253 3.843	108	590	19.8
6:30	4.848 4.438	107	716	22.6
7:00	5.527 5.117 (Mach 15)	105	858	26.8
7:30	6.330 5.920	104	1021	27.3
8:00	7.148 6.738	103	1208	24.7
8:34	7.888 7.478 (Mach 22)	105	1444	
-MECO (Main Engine Cut Off) 8:52--------	**ET SEP (external tank separation)**			

Note: Check the actual data base shown in IL 35.

Negative Return: Point beyond which it is not possible to return to launching area.

Mach 1 = 340 m/s

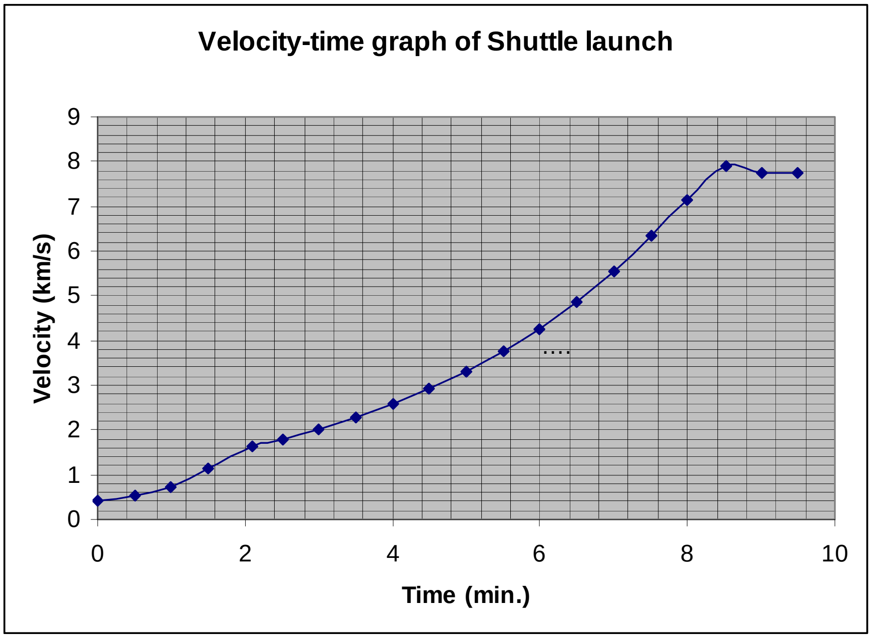


Fig. 35: Velocity-Time Graph of the Launch

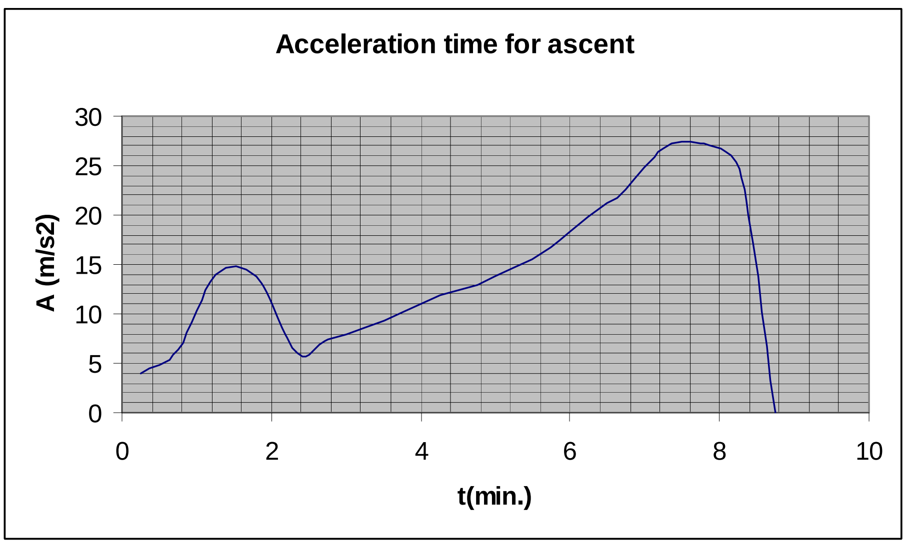


Fig. 36: Acceleration-Time Graph for Launch

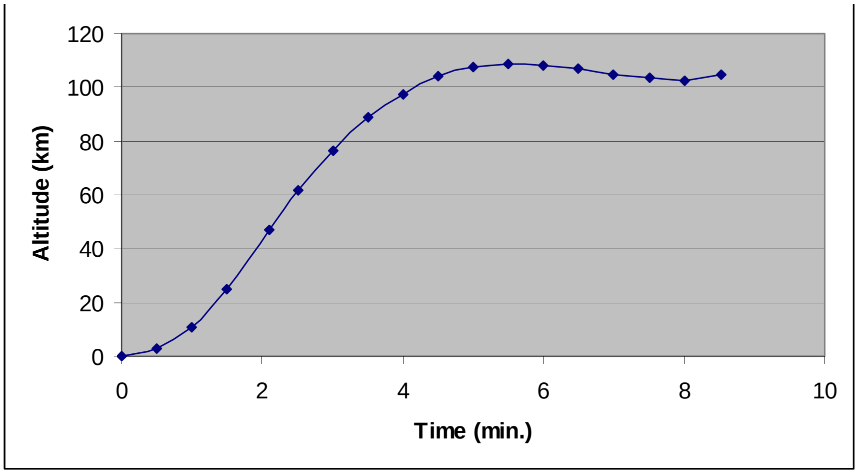


Fig. 37: Altitude-Time Graph of the Launch

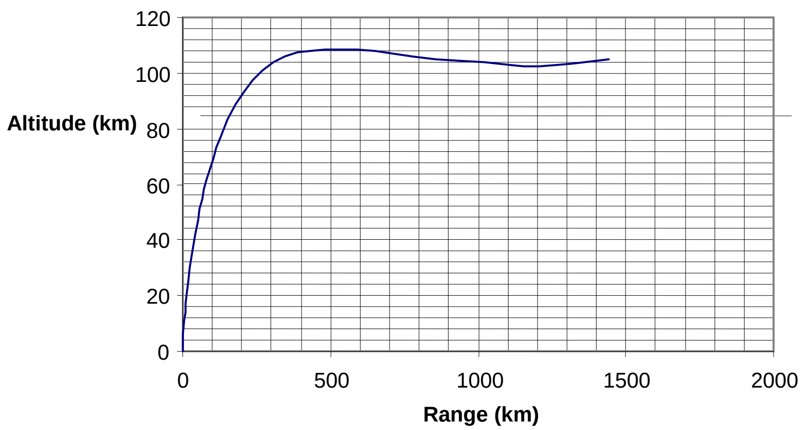


Fig. 38: Altitude-Range Graph of the Launch

Questions and Problems

1. It will be important to know the *vertical* and the *horizontal* velocities and how these change with time. Add two columns to the table above to record the vertical and horizontal velocities for the given times. Remember, however, that you can only find the "average" vertical and horizontal velocities from the given data. Why? Brief note on how to find the "average" vertical and horizontal velocities:

Suggested Table

time (s)	**speed (km/s)**	**height (km)**	**Hor. Dist. (km)**	**Vert. Vel. (km)**	**Hor.Vel. (km)**

2. You are now ready to plot a graph of *vertical velocity against time* and a graph *horizontal velocity against time*. As before connect your points to show a smooth curve. Before plotting these points *predict* what this graph will look like.

   Suggestion: Use the Excel program in your computer to plot these graphs

3. You should now be able to show what the flight path is *as seen from the ground* from far away, *perpendicular* to the plane of flight. If you have watched the launching of any of the flights on TV you know that the flight path looks something like this the flight shown in Fig 28.

Answering Questions Based On the Graphs

1. Using the graphs you have just plotted allows you to answer these important questions:
   a. At what time does the shuttle reach a speed of Mach 1.0? What is the maximum speed the shuttle in Mach units?
   b. Approximately where on the speed-time graph is the acceleration the greatest? Where is it the least? What are these values, expressed in g's? Explain.
   c. At what time do you have maximum and minimum vertical and horizontal acceleration?
   d. Maximum acceleration of about 3 g's occurs toward the end of the launching period, at about 8 minutes (see appendix). Check this on your graph. The effect of g forces on your body will be discussed in the next section.
2. You can estimate the actual distance travelled at the time of MECO? Estimate the actual distance travelled at the time of ET SEP. What do you think the % error of your estimate is? Give reasons for your answer.
3. The rotation of the earth can be used to save energy when the Shuttle is launched. Describe how (in what direction) the Shuttle should be launched and how much energy you could save. First show that the speed of rotation of the earth at the equator is 465 m/s

(about 1600 km/h) and then calculate the speed at the latitude at which the Shuttle is launched. (The Shuttle is launched at Cape Canaveral, a latitude of about 28º N. This speed was added to the speed of the Shuttle in the graph)

In this section we will discuss the forces involved in launching a satellite, specifically the Space Shuttle. Special attention is paid to orbital motion.

Specs for the Space Shuttle

See IL 36 for these specifications.

IL 36 **** Source of Space Shuttle specifications

Orbiter

Length:	37.24 m
Wingspan:	23.79 m
Height:	58.58 ft (17.25 m)
Empty Weight:	69,586.6 kg
Gross Liftoff Weight:	109,000 kg
Maximum Landing Weight:	104,000 kg
Main Engines:	Three Rocketdyne Block 2 A SSMEs,
Thrust (see level):	1.75 MN, each = 5.25×10^6 N.
Maximum Payload:	25,061.4 kg
Fuel ejection rate:	1400 kg/s
Fuel ejection velocity:	3600 m/s
Effective area:	13 m^2
Payload capacity:	22,700 kg

External Tank Specifications (for SLWT)

Length:	48.9 m
Diameter:	10.4 m
Propellant Volume:	2,030,000 L
Empty Weight:	26,560 kg
Gross Liftoff Weight:	757,000 kg
Effective Area:	85 m^2
Burn time:	480s
Liquid Fuel:	Liquid hydrogen and oxygen

Solid Rocket Booster Specifications

Length:	45.6 m
Diameter:	3.71 m
Empty Weight (per booster):	63,273 kg)
Gross Liftoff Weight (per booster):	590,000 kg
Mass of solid fuel:	901020 kg
Fuel ejection rate:	8400 kg/s
Fuel ejection velocity:	2800 m/s
Thrust (sea level, liftoff):	12.5MN
Effective Area (per booster):	11 m^2
Burn time:	120 s
Solid Fuel:	Aluminum and an oxydizing agent

System Stack Specifications

Gross Liftoff Weight:	2.04×10^6 kg
Total Liftoff Thrust:	3.02×10^7 N

Table 3, below, summarizes the data we need for our calculations.

Table 3

First Stage Ascent **SRB engines (2)**	**Second Stage Ascent** **ET (External Tank)**	**Orbit Stage** **Orbiter**
Empty Weight (per booster): 63,273 kg)	**Empty Weight**: 26,560 kg	**Empty Weight**: 69,586.6 kg
Gross Liftoff Weight (per booster): 590,000 kg	**Gross Liftoff Weight**: 757,000 kg	**Gross Liftoff Weight**: 109,000 kg
Effective Area (per booster): 11 m^2	**Effective Area:** 85 m^2	**Main Engines**: Three Rocketdyne SSMEs,
Burn time: 120s	**Burn time**: 480s	**Burn time:** 1250s
Mass of solid fuel: 901020 kg	**Mass of Liquid Fuel:** 730440 kg	**Liquid Fuel:** Liquid hydrogen and oxygen
Fuel ejection rate: 8369 kg/s	**Fuel ejection rate**: 1378 kg/s	**Fuel ejection rate**: 1378 kg/s
Fuel ejection velocity: 2817 m/s	**Fuel ejection velocity**: 3636 m/s	**Fuel ejection velocity**: 3636 m/s
Thrust (sea level, liftoff)**:** 1.25×10^7 N, each	**3 SSME engines Total Thrust:** 5.25×10^6 N	**Total Thrust**: 5.25×10^6 N.

The Effect of Drag on the Shuttle

So far we have neglected the drag on the Shuttle by the atmosphere. Looking at the graph in Fig. 7 we see that the density of the atmosphere at 10 km is about 30% of the sea level density (1,29 kg/m^3) and at about 20 km it is less than 10%. In fact, if you go from ground to higher altitudes, every 5500 meters the density of the air halves. This means in 5500 m altitudes we have only half of the molecules in the same volume of air (for example 1 L) than at sea level. The air becomes continually thinner and lighter. So we can set up a table of values, as follows in table 4.

Table 4

h (m)	**ρ** (ratio of density at earth's surface)	**ρ** (kg/m^3)
0	1	1. 29
5500	1/2	0.65
11,000	1/4	0.32
16,500	1/8	0.16
2,000	1/16	0.080
27,500	1/32	0.041
33,000	1/64	0.020
38500	1/128	0.010
44000	1/256	0.0050
49,000	1/512	0.0025
100,000	1/130,000	1.0 x10^{-6}

We will look at the drag for the heights that correspond to t = 30s, t = 124 s, and t = 480 s. Using the drag equation and the data for total (frontal) area of the Shuttle, we have:

for t = 30s:

h = 2710m, v = (724 – 410) m/s = 314 m/s, ρ = 1.10 kg/m^3

for t = 124 s (just before SRB separation)

h = 47000 m, v = (1633 – 410) m/s = 1223 m/s, ρ = 0.003 kg/m^3

for t = 480s (just before external tank separation):

h = 100,000 m, v = (7148 -410) m/s = 6738 m/s, ρ = 1.0x1.0 x10^{-6} kg/m^3

Using the drag equation $\mathbf{D = \frac{1}{2} A \rho v^2 C_D}$ we find the following values for the drag:

for t = 30s: D = 8.8x10^5 N

for t = 124s: D = 2.24x10^5 N

for t = 480s: D = 2300 N

Comparing these values to the thrust:

for $t = 30s$ $T = 1.25x10^7$ N or **$D / T = 0.070$**, which is 7 % of the thrust.

for $t = 124s$ $T = 1.25x10^7$ N or **$D / T = 0.018$**, which is about 2% of the thrust.

for $t = 480s$ $T = 3.50x10^6$ N or **$D /T = 0.0007$,** which is about 0.07% of the thrust.

The effect of drag then is nearly negligible after the SRB separation.

Of course, when the Orbiter returns on re-entry and is moving through the atmosphere at lower heights at about 7 km/s, we will see that the drag then is sufficient to raise the temperature of the surface of the Orbiter to over 1000 degrees Centigrade.

To find the acceleration at any time neglecting drag

We saw earlier that the net force required to accelerate the Shuttle is given by

$$\mathbf{F_{net} = T_{net} - W) = m_t\, a}\text{, then}$$

$$\mathbf{a = F_{net} / m_t = \frac{[T_{net} - W]}{m_t}}$$

Since $\mathbf{w = (m_i - Rt)g}$, the acceleration (instantaneous) then becomes:

$$\mathbf{a = \frac{[T_{net} - (m_i - Rt)g]}{m_i - Rt}}$$

This looks like a very complicated equation but it really is quite straight-forward.

We can now calculate the instantaneous accelerations for any time t between 0 and 480 s. We will calculate the values for t = 30s, t = 125s and t = 480 s.

You must remember (see height-range graph above) that at t= 30s the Shuttle is almost vertical but at t=215s, the angle of ascent is about 45°, and for t=480s the Orbiter is moving horizontally.

According to our a-t graph, the acceleration at t=30 is about m/s^2, for t=125s it is about 16 m/s^2, and for t=280s, it is about 27 m/s^2. We want to see if the dynamics approach gives a similar answer.

To calculate the acceleration for t = 30s

We know that during the first 125 seconds the combined thrust of the ascent is: $3.02x10^7$ N. The ejection rate for the SRB engines is 8369 kg/s and that of the Orbiter engines is 1378 kg/s. The initial mass $\mathbf{m_i}$ is $2.04x10^6$ kg, and the final mass is:

$$\mathbf{m_i - Rt =} 2.04x10^6 \text{ kg} - (8369 \text{ kg/s x } 30 + 1378 \text{ kg/s x}30) = 1.74x10^6 \text{ kg.}$$

Therefore

$$\mathbf{a = (}3.02x10^7 \text{ N} - 1.74x10^6 \text{ kgx}9.8) / 1.74x10^6 \text{ kg} = 7.3 \text{ m/s}^2$$

Of course, this rate will be a little high for t=30s, because the Shuttle reduces the thrust to 75% between t = 20s and t = 45s. So we get .75x7.3 = 5.5 m/s^2, which agrees with the kinematic calculation of the average velocity. We must also remember that Shuttle at his time is inclined to the vertical to about 70º.

<u>To calculate the acceleration for t = 105s</u> (before SRB separation)

As before: The ejection rate for the SRB engines is 8369 kg/s and that of the Orbiter engines is 1378 kg/s. The initial mass $\mathbf{m_i}$ is 2.04×10^6 kg, and the final mass is:

$$\mathbf{m_i - Rt =} 2.04 \times 10^6 \text{ kg} - (8369 \text{ kg/s x } 105 + 1378 \text{ kg/s x} 105) = 1.02 \times 10^6 \text{ kg.}$$

Therefore

$$\mathbf{a = (}3.02 \times 10^7 \text{ N} - 1.02 \times 10^6 \text{ kg.kgx}9.8) / 1.02 \times 10^6 \text{ kg} = 19.6 \text{ m/s}^2$$

<u>To calculate the acceleration for t = 480s (8:00 minutes)</u>

By the time the Shuttle reaches 8 minutes, the trajectory is almost level (see graph above) and therefore we can write the acceleration equation as

$$\mathbf{a} = \frac{\mathbf{T_{net}}}{\mathbf{m_i - Rt}}$$

The Orbiter is now being accelerated by the Orbiter's three main engines. The mass of the Orbiter is about 100,000 kg. Therefore the acceleration, on full thrust, could be

$$a = 5.25 \times 10^6 \text{ N} / 1.0 \times 10^6 \text{ kg} = 52.5 \text{ m/s}^2$$

This is more than 5 g's and since acceleration must be kept under 3 g's, the thrust must be reduced to less than 60%.

Note: The acceleration calculated by using $\Delta v / \Delta t$ from values taken using the velocity-time graph, are only average values. See the data given by NASA (IL 35) to convince yourself that our results are pretty good.

Finding the Velocity of the Shuttle Using the Rocket Equation

We have found that if the drag is small or negligible, and we can ignore the change of gravity, we can calculate the velocity at time t using the rocket equation in a constant gravity:

$$\mathbf{v = v_e \ln (m_i / m_f) - gt}$$

However, we have thrust contributions from two sources for the first stage ascent: from the two booster rockets as well as from the external tank feeding the three SSME engines. Therefore the rocket equation becomes: $\mathbf{v = v_{eB} (\ln (m_i / m_{fB}) - v_{eT} \ln (m_i / m_{fO}) - g\, t}$ where $\mathbf{m_i} = 2.04 \times 10^6$ kg total, initial mass of the Shuttle, $\mathbf{m_{fB}}$ is the final mass after the burnout of the booster fuel and $\mathbf{m_{fO}}$ is the final mass contributed by fuel consumption of the SSME, and $\mathbf{g}$ the effective mean value of the gravitational acceleration for the portion of the flight considered. (Note: $\mathbf{g_{at}}$ 100 km height is only about 3% lower than g on the surface of the earth). $\mathbf{v_{eB}}$ is the ejection velocity of gases for the boosters and $\mathbf{v_{eO}}$ is the ejection velocity of the gases for the SSMEs of the Orbiter.

We can then write the equation this way:

$$\mathbf{v = v_{eB} \ln [(m_i / (m_i - R_B\, t)] + v_{eO} \ln [(m_i / (m_i - R_O\, t)] - g\, t}$$

We will calculate the velocity of the Shuttle at the end of the ascent stage when t = 2:05, or t = 125s. According to NASA data the inertial velocity then is 1.633 km/s, and the velocity relative to the earth's surface 1.633 – 0.410 = 1233 m/s.

We know that:

$v_e = 3600$ m/s, t = 125s, $m_i = 2.04 \times 10^6$ kg, $m_i - R_B\, t = = 2.04 \times 10^6 - 8369 \times 125 = 9.94 \times 10^5$ kg.

and $m_i - R_O\, t = 2.04 \times 10^6 - 1378 \times 125 = 1.87 \times 10^6$ kg.

Substituting we have:

$$v = 2817 \times \ln [2.04 \times 10^6 / 9.94 \times 10^5]. + 3636 \times \ln [2.04 \times 10^6 / 1.87 \times 10^6 \text{ kg}] - 9.8 \times 125$$

$$v = 3600 \ln (2.04) - 9.8 \times 125 = 1138 \text{ m/s.}$$

The velocity of the Shuttle at this time is (relative to the earth) is actually 1223 m/s. So our calculation, based on a simple approach, seems to work quite well, with an error of less than 10%.

Stage 2: From t =125s to t = 510 s, from h = 47 km to h = 105 km.

The Shuttle has lost its two boosters and is propelled by the 3 SSMEs of the Orbiter, drawing liquid fuel from the External Tank. The rocket equation for this stage then becomes:

$$\mathbf{v_2 = v_{eO} \ln [(m_i / (m_i - R_O\, t)] - g\, t}$$

(Note that the Orbiter is flying almost horizontally during stage 2 and therefore we can neglect the gt effect, at least for an approximate solution**.)**

$m_i = 2.04 \times 10^6$ kg – (mass of the two SRBs plus the fuel lost during stage 1):

2.04×10^6 kg – (1.18×10^6 kg + 1.72×10^5 kg) = 6.90×10^5kg

m_f: essentially the fully loaded Orbiter, 109000 kg. Substituting into our equation we get 6710 m/s. The final velocity at the end of stage 2, then, is the velocity at the end of stage 1 plus the velocity calculated for the end of stage 2.

Therefore we have: 1138 m/s + 6710 m/s = 7848 m/s (Relative to the earth). This compares well to the velocity reported as 7478 m/s (relative to the earth).

Problems and Questions for the Student

1. What acceleration do the astronauts experience at the very beginning? There is no significant mass loss at this stage.
2. What is the acceleration just after SRB SEP? Just after MECO ?
3. Refer back to the v-t graph in section III and check these values. Comment.
4. Using the speed-time graph you plotted in the previous section plot a force-time graph for the motion up to ET SEP. To save time simply write the values of the force in terms of g (10 m/s^2).
5. What is an 80 kg astronaut's "weight" at maximum acceleration? Remember the Shuttle is flying at an angle.

Calculating the Height Using the Rocket Equation

The following is presented as an advanced problem for students who have taken elementary calculus.

Using the equation to calculate the velocity we can find an equation to predict the height to which a rocket rises in a vertical trajectory.

Using the equation for the velocity at time t, when the rocket start from rest in a gravitational field:

$$\mathbf{v = v_e \ln [(m_i / (m_i - R\, t\,)] - g\, t}$$

But we know that $\mathbf{v = dh / dt}$

Therefore we can write:

$$\mathbf{dh = v_e \ln [(m_i / (m_i - R\, t\,)] \, dt - g\, t\, dt}$$

Integrating both sides we have

$$\mathbf{\int_0^h dh = \int_0^t v_e \ln [(m_i / (m_i - R\, t\,)] \, dt - \int_0^t g\, t\, dt}$$

Integrating by parts you can show that

$$\mathbf{h = v_e\, t + [(m_i - R\, t\,) / R]\, (m_i - R\, t\,) / m - \tfrac{1}{2}\, g t^2}$$

Going back to our original launch problem of the small vertically moving rocket, calculate the height of the package at the end of burn-out time. Compare this value with the one you obtained using graphical methods. Comment.

A Problem of Controlled Explosion

1. SRB SEP is accomplished by explosive devices called separation rocket motors. There are eight of these, each capable of developing a thrust of 98000 N for a duration of .5 seconds. What will be the relative velocity of separation?

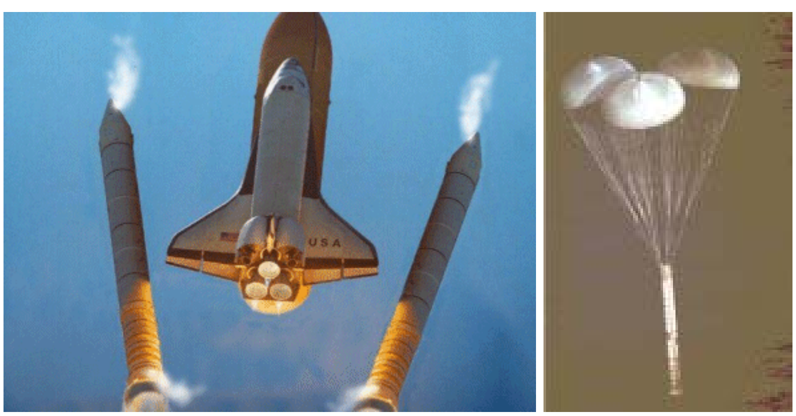


Fig. 39: The Boosters Are Discarded At ≈2 Minutes into the Flight and Later Recovered

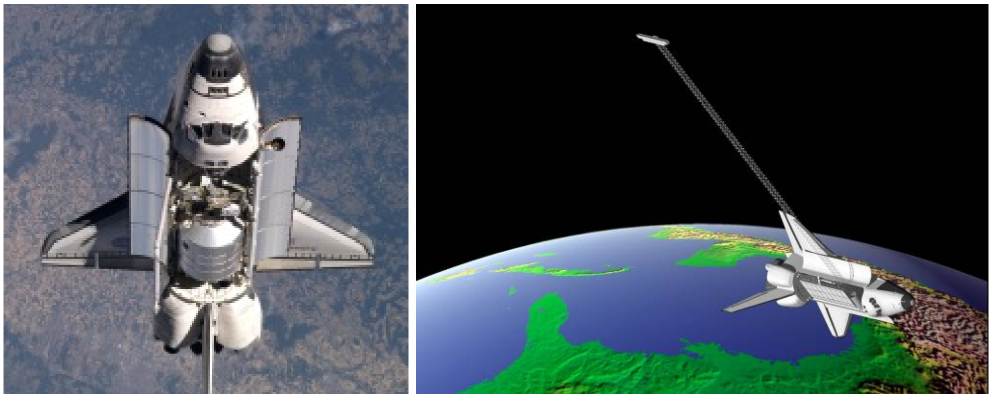


Fig. 40: The Orbiter Settles Into In a Stable Orbit

IL 37 ** Overall Orbiter specs (Source of Fig. 42)

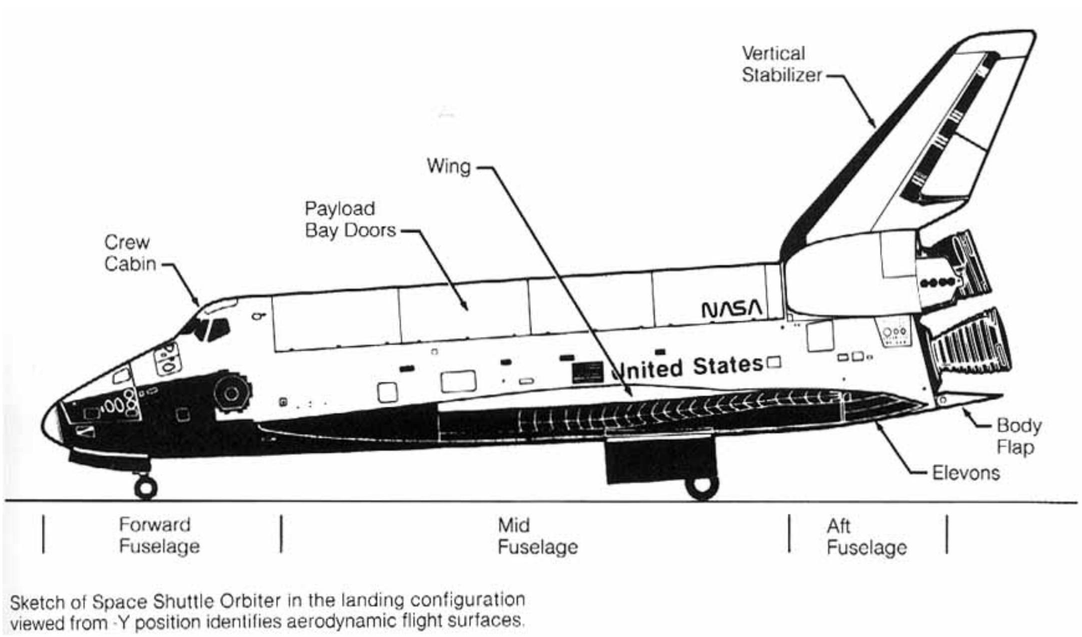


Fig. 41: Sketch of Space Shuttle Orbiter in the Landing Configuration Identifies Aerodynamic Flight Surfaces

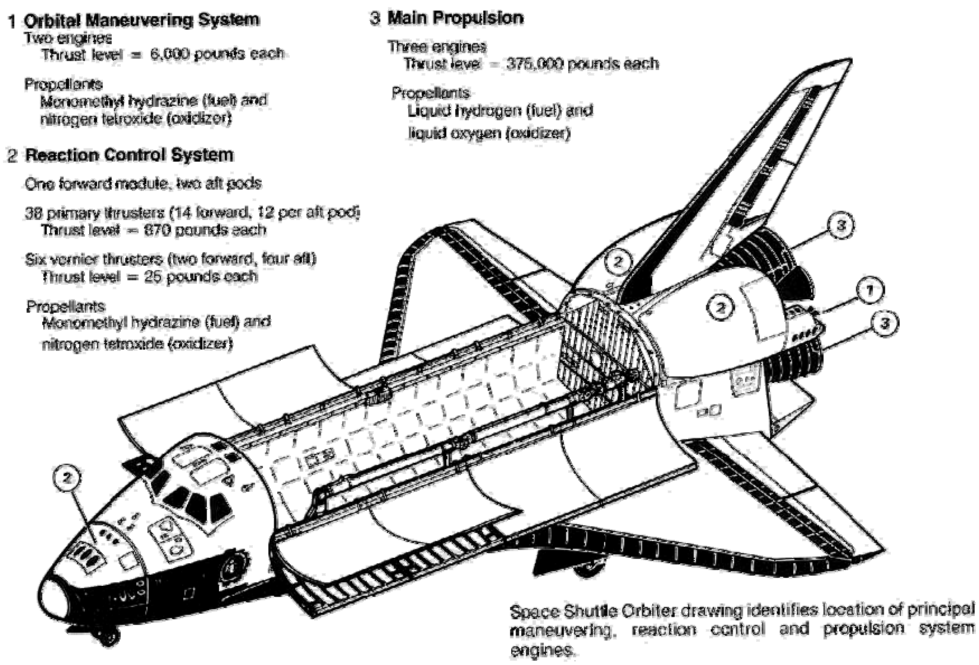


Fig. 42: Detail of the Orbiter Open. The Orbiter Is Always Open When In Orbit.

The Final Phase

1. The Shuttle finally settles into a near-circular orbit of 160 x 161 nautical miles.
   a. What is the period of the Shuttle in this orbit ?
   b. What is the speed of the Shuttle in this orbit ?
   c. What is the value of the gravitational field at this altitude?
   d. We often refer to this state as the state of "weightlessness". Does it really make sense to say that in the light of the fact that the gravitational field strength is still quite high? Discuss.

Some Hypothetical Situations

1. It is interesting and instructive to consider such hypothetical situations as the following. A small rocket is fired from the Shuttle in the opposite direction to its motion. Describe the motion of the rocket if it leaves the Shuttle with a speed relative to the Shuttle
   - of 5000 m/s?
   - of 8000 m/s, i.e. equal to the Shuttle's orbital speed.

   What is the "weight" of the rocket at the moment it leaves the Shuttle? What is the weight of the Shuttle?

2. A NASA handbook gives the equation for the orbital velocity as

$$\mathbf{v} = \frac{\mathbf{G[M_e + M_g]}}{\mathbf{(R_e + h)}}$$

where G = 6.67 x 108 ; M_e (Mass of Earth) = 5.97 x 10^{24} kg; M_s (Mass of Shuttle) = 1.36 x 108 g; R_e (Radius of Earth) = 6.378 x 10^6 m; and h = orbital altitude. Compare this formula with Kepler's formula discussed earlier. Check this formula by both a numerical calculation and by direct derivation. Comment. How could this formula be simplified to give a reasonably good answer? Explain.

Finding the Mass of a Planet

In order to find the mass of a planet we must know the period of a satellite that is orbiting it, as well as the average distance.

a. Calculate the mass of the Earth from the satellite data only. The value of G, of course, must be known.
b. Find the mass of the moon if a satellite in circular orbit has a period of 3.97 hours. The radius of the moon is 3.47x10^6 m. The satellite is 5000 meters above the surface.

Changing Orbits

1. Before manoeuvring into a near-circular orbit the Shuttle is pushed into an elliptical orbit with an apogee, perigee of 156 x 35 nautical miles.
   a. What is the period of the Shuttle in this orbit?
   b. What are the maximum and minimum speeds of the Shuttle?
   c. Why is this not a desirable orbit?
2. The pilot of the Shuttle wishes to manoeuvre into a lower orbit, from a circular orbit at altitude of 250 km to a circular orbit at an altitude of 200 km. What directional and speed adjustments would you have to make? Discuss.
3. The shuttle is trying to connect to a space station that is circular, at a distance of 160 nautical miles. The Shuttle is in the same orbit but 100 nautical miles "behind". As a navigator officer you are to decide on a course of action. What would you recommend?

An Elliptical Orbit

We will conclude our study of a Shuttle mission with the placing of a small 100 kg satellite into an elliptical orbit. The satellite is placed by launching it from the Shuttle while it is in a near-circular orbit at an altitude of 250 km. The satellite is to be placed in an elliptical orbit with perigee altitude of 250 km and an apogee altitude of 500 km.

a. What must be the "injection" velocity?
b. What is the speed at apogee?
c. Find the period of the satellite.

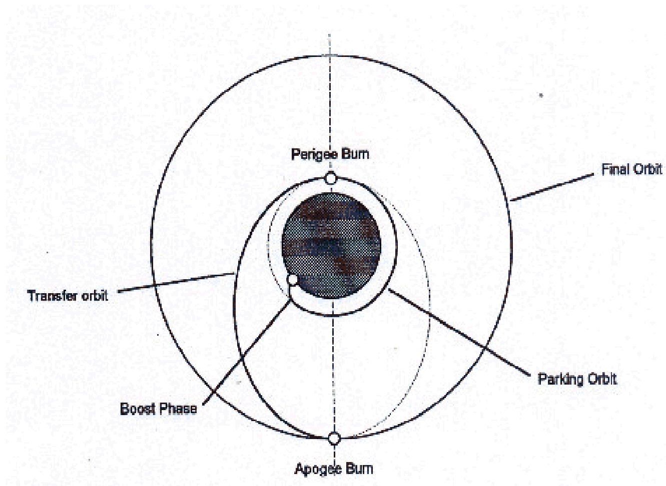


Fig. 43: An Elliptical Transfer Orbit

The Shuttle Returns to Earth

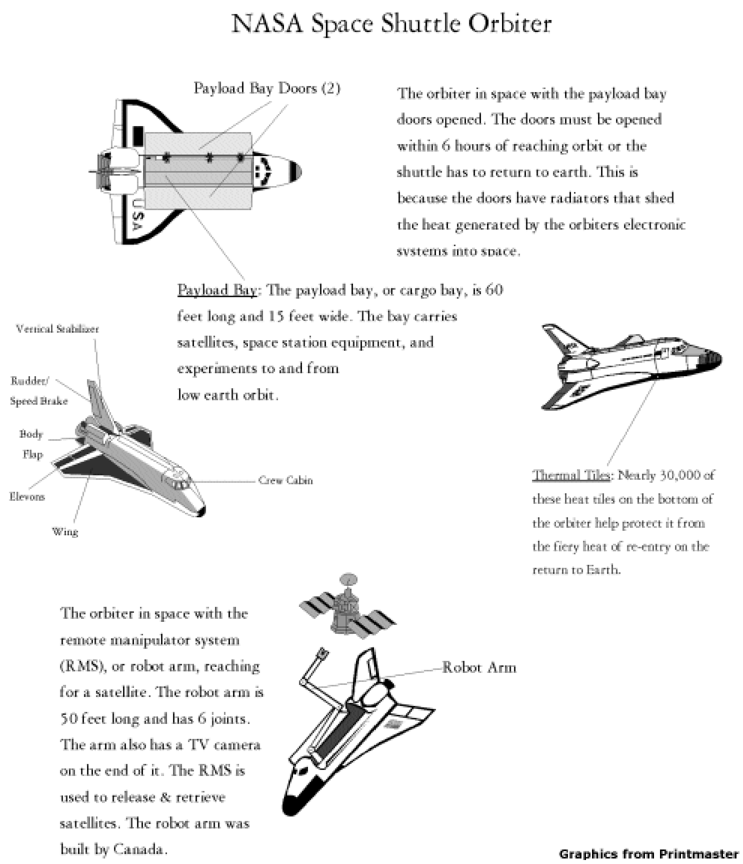


Fig. 44: The Re-entry of the Orbiter

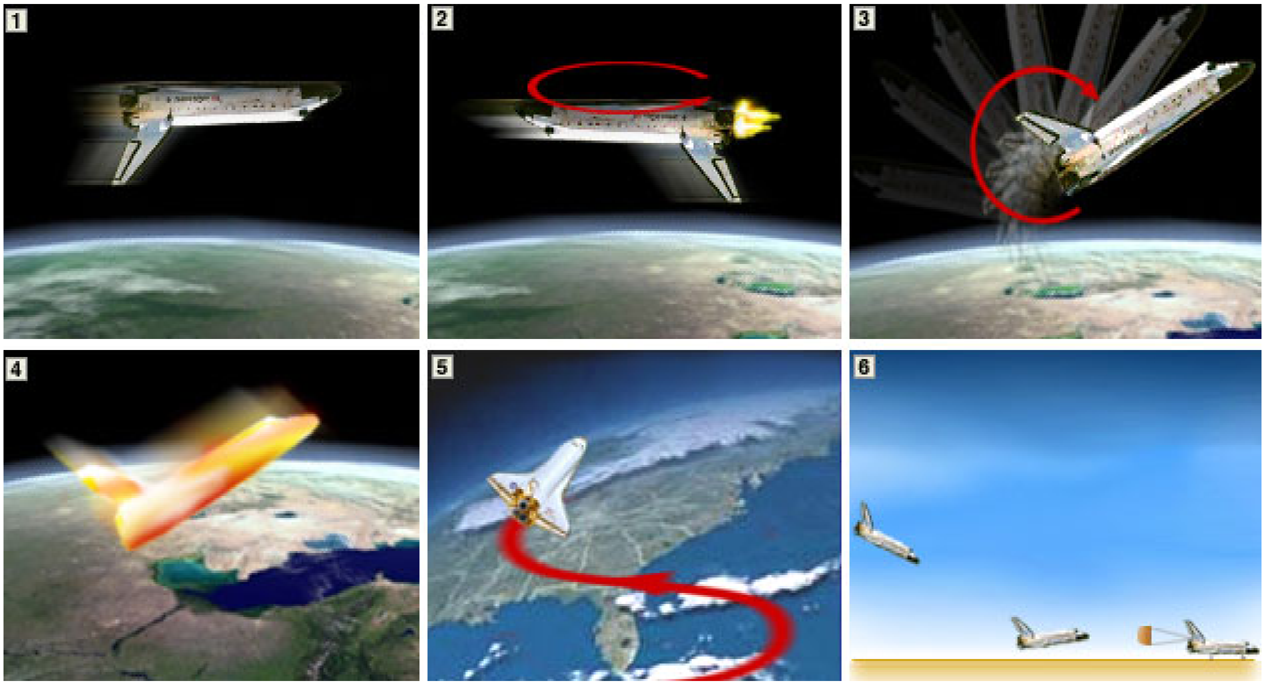


Fig. 45: The Orbiter Re-entry Sequence

To return to Earth the space shuttle must make a series of complicated maneuvers to align itself into the correct position to achieve a safe descent.

1. The shuttle flies upside down in orbit to control its heating.
2. To re-enter the atmosphere, the shuttle is turned tail first to the direction of travel, and fires its engines to slow its speed.
3. The orbiter is then flipped the right way up and enters the top layer of the atmosphere at about a 40-degree angle from horizontal with its wings level.

Fig. 46: An Artist's Sketch Showing The Heat Being Built And Dissipated Around The Shuttle As It Re-Enters The Atmosphere

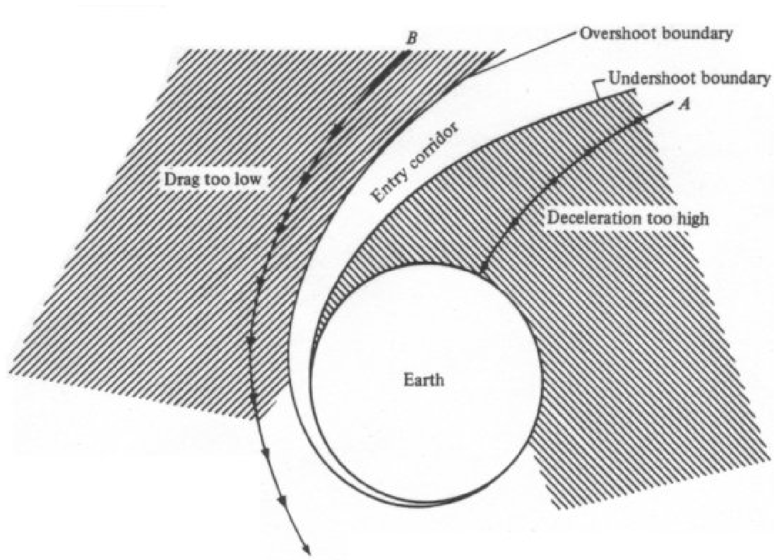


Fig. 47: The Reentry Maneuver through the Window Called "Entry Corridor"

Any object re-entering the atmosphere of Earth or that of another world must do so within a very small range of angles in order to reach the surface successfully. The upper and lower limits on this re-entry region are determined by a combination of three factors: the trajectory of the object, its rate of deceleration, and aerodynamic heating. We will discuss the importance of each of these issues in turn. The narrow region dictated by these parameters is known as the entry corridor illustrated in Fig. 47, above.

The trajectory a vehicle follows when returning to Earth depends in part on the type of orbit the object traveled in order to reach the planet. The orbital path is significant because it determines how fast the vehicle is traveling when it first encounters the atmosphere. One special type of orbit is the circular orbit that approximates the path most spacecraft like satellites and the Space Shuttle follow while orbiting the Earth. These vehicles typically circle the Earth at speeds between 17,000 and 18,000 mph (27,360 to 28970 km/h) and will re-enter the outer fringes of the atmosphere at these high speeds.

Table 4

Time (min)	Forward Velocity (km/s)
0	7.672
3.0	7.562
5.0	7.500
10.0	6.720
13.0	5.880
14.0	5.600
15..0	5.180
17.0	4.340
18.0	3.920

19.0	3.360
20.0	2.800
21.0	2.280
23.0	1.400
24.0	1.060
24.5	.857
27.0	3430
30.0	.0950
32.0	0

Distance/Time Graph

Re-entry and Landing of the Space Shuttle Orbiter

Table 5

Time (min)	(km) Altitude
0	122
5	95
10	70
14	62
18	55
20	45
23.0	32
24.0	27
24.5	25
27.0	15
28.0	4
29.5	0.6
29.7	0.041
29.8	0.027
30.0	0

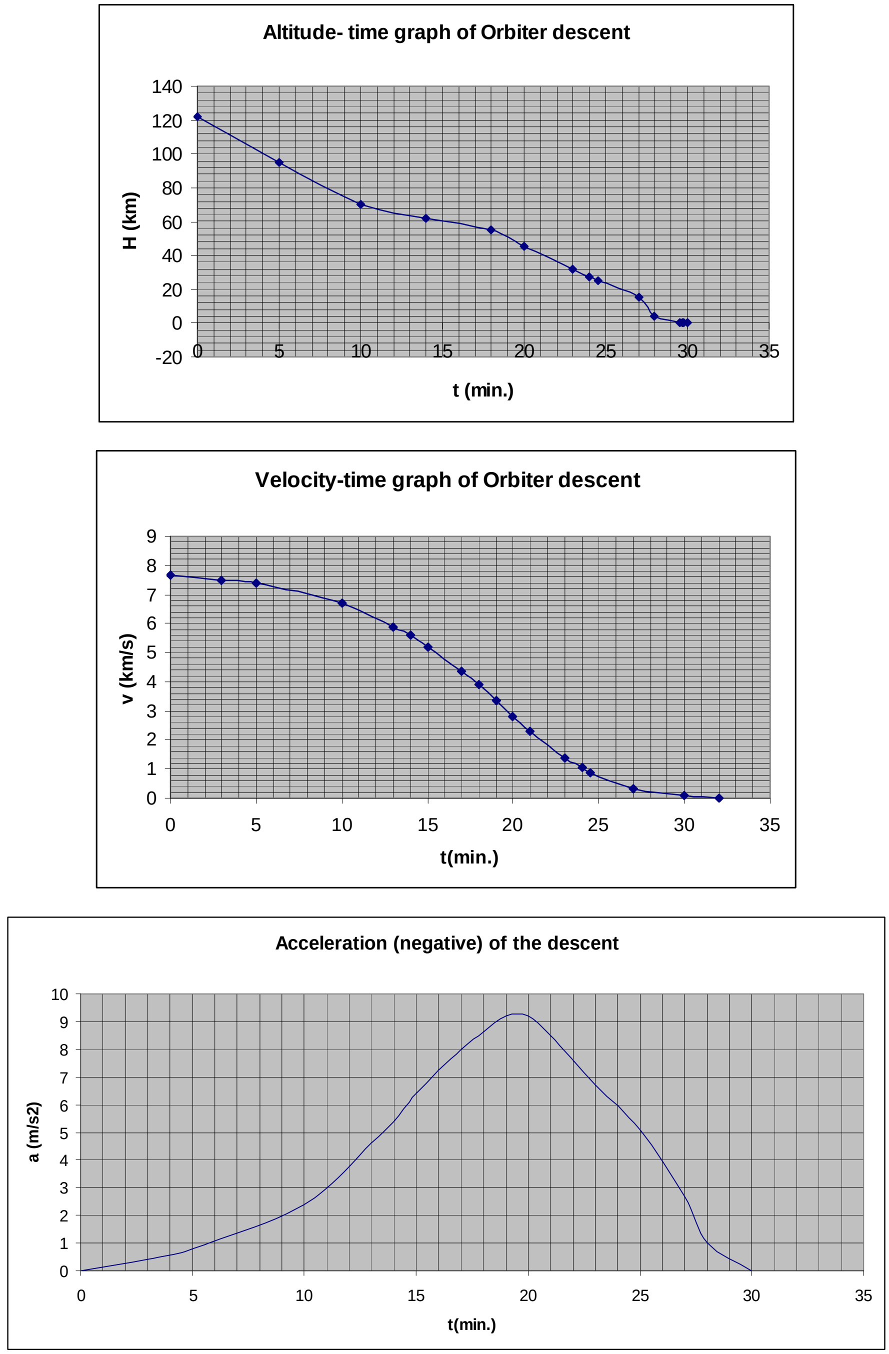


Fig. 48: h-t, v-t, and a-t Graphs of Orbiter Decent

IL 38 **** A very comprehensive history of the Space Shuttle

The following description of the re-entry of the Orbiter is taken from IL 39, below:

IL 39 **** An excellent description of the Orbiter reentry

At the start of re-entry, hot ionized gases surround the shuttle. Throughout the decent the friction between the shuttle and the earth's atmosphere create even more and more heat beneath the orbiter. The black ceramic tiles located on the bottom and along the wing tips and nose of the orbiter protect it from the nearly 1,648 degrees Celsius (3000 degrees F) temperatures. To rid the orbiter of some of this heat, it makes a series of small rolls from left and right throughout its fiery fall.

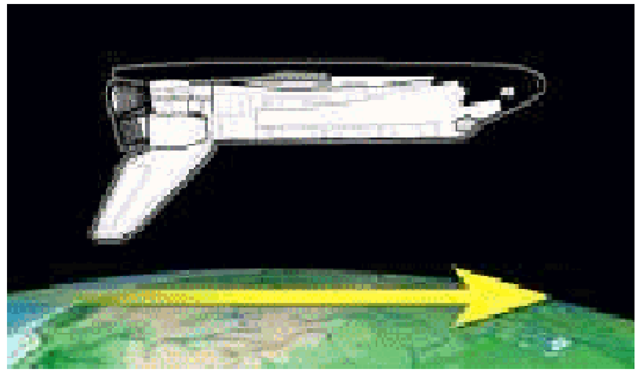


Fig. 49

During the Apollo and earlier missions into space, the build-up of hot gases beneath the spacecraft prevented communication between the astronauts and mission control on earth. Called a "blackout", it lasted for about six minutes. For many controllers, it was the longest six minutes of their lives. Communication with the space shuttle, however, is maintained during its descent. Instead of signals going downward to earth, they are sent upward to a satellite and then relayed to mission control.

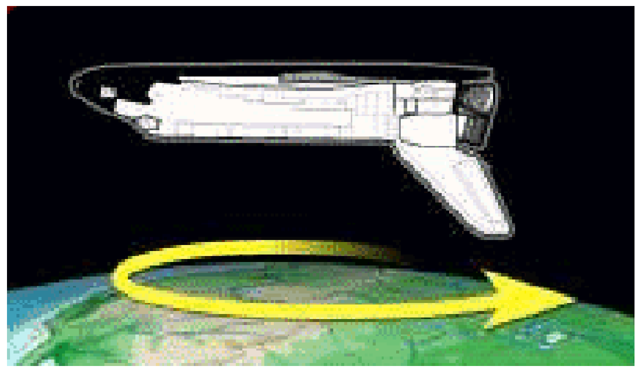


Fig. 50

Most of the orbiter's descent is controlled not by the pilot or the commander, but by its onboard computers. The computer uses information on air speed and air pressure to make the slight adjustments necessary to maintain the orbiter in a proper glide path through the ever thickening atmosphere. These moves are done by moving the elevons (the combination of elevators and ailerons on ordinary airplanes) along the trailing edge of the wings, the rudder on the tail and the body flap located beneath the main engines at the rear of the orbiter. Although the commander and mission control are carefully monitoring the flight, the commander doesn't take control of the orbiter until it has slowed to Mach 1. This occurs within visual range of the landing strip just 40 kilometres (25 miles) away.

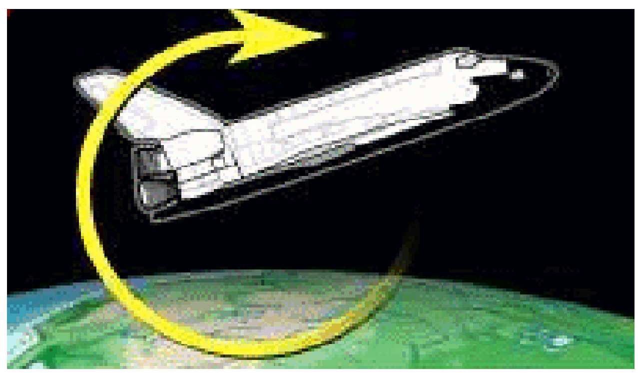


Fig. 51

Fig. 52

Finally it should be mentioned that as the vehicle slices through the atmosphere at velocities greater than *the speed of sound, the sonic boom thunders across the Florida landscape, heralding the Space Shuttle's return. The sound is actually two distinct claps produced by compressed air in front of the nose and wings, which create shock waves that spread away from the aircraft.*

The Space Shuttle orbiter's main landing gear touches down on the runway at 213 to 226 miles per hour. As the nose pitches down and makes contact with the runway, a 40-foot drag chute is deployed from the vehicle's aft end, and the orbiter rolls to a stop.

Based on the above description, the most difficult and dangerous part of the journey for the astronauts is the re-entry phase. To study this phase more closely we well look at the following table which is taken from an actual re-entry. The questions and problems that follow are based on the data given in the table. L stands for "Landing minus hr:min:sec."

IL 40 *** The descent of the Orbiter

A Description of the Re-entry Relevant To the Understanding of the Physics of Re-Entry

Taken from IL 41

IL 41 *** Description of the reentry of the Orbiter

The Physics of Space Shuttle (Orbiter) Re-Entry

At 05.12 Pacific Coast Time on 9 August, 2005, the Space Shuttle orbiter Discovery rolled to a stop on the runway at Edwards Air Force Base in California. The first shuttle mission for thirty months had been completed safely. If Mission STS-114 had flown according to its original schedule, there would probably have been little media attention. Instead, people all over the world were watching, and breathing a collective sigh of relief.

The anxiety provoked by Discovery's homecoming was stimulated by the tragedy that overtook its predecessor, Columbia. The last hour of any shuttle mission represents an extreme challenge to pilot and engineer alike. This Entry is an attempt to explain that challenge.

The Fundamental Problem in Re-Entry

The phase of a spaceflight during which the craft leaves earth orbit and descends through the upper atmosphere is generally known as 're-entry'. In order to be in stable earth orbit in the first place, the craft must have attained and maintained a critical velocity. This orbital velocity is nearly thirty times the speed of sound - around 13 kilometres per second. If the craft moves any more slowly than this, it will descend to a lower orbit under the influence of gravity. Because the craft will now encounter atmospheric resistance, it will lose energy and fall to earth.

In order to make a safe landing, a returning spacecraft has to lose nearly all of that orbital speed. The operation is basically a reversal of the launch phase, and this means that the returning craft must sink as much kinetic energy[1] *as the propulsion systems generated between lift-off and orbit. Theoretically speaking, there are four fundamentally different methods of doing this:*

- *Powered Deceleration*
- *Energy Exchange*
- *Mass Shedding*
- *Energy Dissipation*

An explanation of the shuttle's methods will be helped by a brief consideration of all four.

Powered Deceleration

This can be achieved by rocket thrust opposed to the direction.This can be achieved by rocket thrust opposed to the direction of motion. The shuttle does this at the very beginning of its re-entry, to trim the speed and so initiate the descent from orbit. The equivalence of re-entry and launch energy means that it's not a practical proposition for the whole descent, however. Using this method would more than double the fuel load required at launch[2], and require the craft to carry half of it while in space.

Energy Exchange

This ingenious braking method requires the conversion of kinetic energy into potential energy, which can then be stored in some kind of separate device. It's ultimately the best solution of the grounds of sustainability: the energy is not wasted and can in principle be used in subsequent spaceflights. The best-known concept of this kind is the space elevator, a theoretically possible machine but one that's far beyond present-day engineering feasibility. Energy exchange is not an option for today's spacecraft, therefore, though it is a serious objective for far-future technological development.

Mass Shedding

This method is conceptually exemplified by a pilot ejecting from his damaged plane - most of the system's energy remains invested in the part that carries on and crashes. Though the idea works, the shuttle program's fundamental premise of orbiter re-usability precludes it. The only mass shed by the shuttle is in the launch phase, when the spent rocket-boosters and empty fuel tank fall back to the sea. If the vehicle was dispensable, the astronauts could transfer into a small escape vehicle which would then be jettisoned from the main craft. The re-entry phase of other space programs (such as Apollo) made use of mass shedding. The shuttle program, though, is bound by the principle that everything that enters earth orbit comes home again.

Energy Dissipation

This method differs from energy exchange in that the kinetic energy is progressively (and wastefully) converted to another form, such as heat, as the descent proceeds. It's the principal method for re-entry braking in all mankind's space programs to date, and for the space shuttle it's the only method, once the descent proper has begun. The amount of energy that must be dissipated is very large. Stopping a one hundred ton craft from a speed of 13 km per second in eighty minutes requires nearly two thousand megawatts of power[3]

Summary of the Reentry of the Orbiter

Time	**Altitude / velocity**	**Description / Density of atmosphere**
L-1:00:00	Orbit (320 km) 27,700 km / h (7756 m/s) Mach 23	$4x10^{-11}$ (kg /m^3). Position Orbiter for entry. Execute de-orbit burn command on computer. The burn should last about 3 minutes,. Though such an orbit can be sustained indefinitely, a speed trim of about 300 kph is sufficient to initiate descent. The outside of the Shuttle heats to over 1,500 °C during re-entry. The vehicle begins re-entry by firing the OMS engines opposite to the orbital motion for about three minutes. The deceleration of the Shuttle lowers its orbit perigee down into the atmosphere. This OMS firing is done roughly halfway around the globe from the landing site
L - 0:30:00 t = 0 min.	121 km 27,400 km/h (7.67 km/s) Mach 23	Atmospheric entry begins $3.5x10^{-8}$ kg/m^3 The vehicle starts to enter the atmosphere at about 400,000 ft (121 km) traveling around Mach 23. The vehicle is controlled, achieved by a combination of RCS thrusters and control surfaces, to fly at a 40 degrees nose-up attitude producing high drag, not only to slow it down to landing speed, but also to reduce re-entry heating. In addition, the vehicle needs to bleed off extra speed before reaching the landing site. This is achieved by performing s-curves at up to 70 degree bank angle. Just before it reaches the upper atmosphere, the orbiter is oriented nose first by some of its steering engines and enters the top layer of the atmosphere at a 40 degree angle. This orientation means the heat shield beneath the shuttle will experience most of the friction with the atmosphere at the beginning stages of re-entry. It also ensures the orbiter does ship off the top of the earth's atmosphere like a flat stone skipping of the surface of the water, and out into space. To reduce re-entry risks further, any remaining fuel in the RCS engines is jettisoned (See Fig.).
L - 0:25:00 t = 5 min.	95 km 26,800 km/h (7.500 km/s) Mach 22	Beginning of partial communications blackout, $2x10^{-6}$ kg/m^3 This is caused by ionized particles enveloping Orbiter. The sheer force of the Space Shuttle (or any other entering object) pushing through the atmosphere at fantastic speed causes the air to be compressed ahead of it, creating a shockwave. This shockwave is so compressed and so hot that

		it becomes a plasma -- the fourth state of matter. Plasma is electrically conductive, and in fact radio waves have a hard time penetrating it; this is why spacecraft can lose contact with the ground during reentry. (The Shuttle cheats; it doesn't talk directly to the ground, but rather to the TDRS satellites in geosynchronous orbit and thus not obscured by the plasma. But it's line-of-sight to the satellites isn't always good, so even the Shuttle's signal may drop out periodically during entry.)
L - 0:20:00 t = 10 min.	70 km 24,000 km/h (6.720 km/s) Mach 20	<u>Maximum heating effect</u>: Nose and wing $9x10^{-5}$ kg/m^3 Plasma, known as the fourth state of matter, does not conform to the gas laws of conventional thermodynamics The formation of the pressure wave, therefore, also creates extreme temperatures. The plasma stream is electrostatically-charged too, and so it concentrates at acute surface contours, a behaviour related to the ground-level phenomenon of corona discharge (St Elmo's Fire). The resultant effect is particularly intense local heating at the airframe's leading edges. Leading edges of Orbiter reach 1500 E C.
L - 0:12:00 t = 18 min.	55 km 13,000 km/h (3.640 km/s) Mach 11	<u>End of partial communication blackout</u> $7x10^{-4}$ kg/m^3 From atmospheric penetration at around Mach[7] 25 or more, the plasma shroud persists down to about Mach 15, whereupon it is replaced by a less energetic though still chemically-reactive flow.
L - 0:10:00 t = 22 min. ________ L - 0:06:00 t = 24 min.	35 km 6,400 km/h (1.80 km/s) Mach 7 ________ 27 km 3890 km/h (1.09 km/s) (Mach 3.2)	0.0082 kg/m^3 <u>Region of highest deceleration</u> (about 7-9 m/s^2). The plasma cloud diminishes at around Mach 7, and the shock wave dissipates at about Mach 3. Below this speed, manual flight becomes routine, although there is still the descent through the sound barrier to come. ________ <u>Deploying air data probes</u> 0.03 kg/m^3 In the lower atmosphere the Orbiter flies much like a conventional glider, except for a much higher descent rate, over 10,000 feet (3000m) per minute (roughly 20 times that of an airliner). It glides to landing with a glide angle of 4:1. At approximately Mach 3, two air data probes, located on the left and right sides of the Orbiter's forward lower fuselage, are deployed to sense air pressures related to vehicle's movement in the atmosphere.

L - 0:03:00 t = 27 min.	15 km 1200 km/h (.340 km/s) (Mach 1.0)	Linng up with runway for final approach 0.20 kg/m^3 When the approach and landing phase begins, the Orbiter is at 10,000 ft (3048 m) altitude, 7.5 miles (12.1 km) to the runway. The pilots apply aerodynamic braking to help slow down the vehicle. The Orbiter's speed is reduced from 424 mph (682.3 km/h) to approximately 215 mph (346 km/h), (compared to 160 mph for a jet airliner), at touch-down.
L - 0:02:00 t = 29 min	4 km (678 km/h)(190 m/s)	Beginning auto-land guidance 0.819 kg/m^3
L - 0:00:30 t = 29min.30s	600 m 560 km/h (157 m/s)	Adjusting glide slope 1.29 kg/m^3
L - 0:00:14 t = 29 min. 46s L - 0:00:00 t = 30 min.	27 m 530 km/h (148m/s)	Deploying landing gear 1.29 kg/m^3 The landing gear is deployed while the Orbiter is flying at 267 mph (429.7 km/h). In additional to applying the speed brakes, a 40 ft (12.2 m) drag chute is deployed once the nose gear touches down at about 185 knots. It is jettisoned as the Orbiter slows through 60 knots.
	0 m 340 km/h (95 m/s)	Touchdown 1.29 kg/m^3 As it aligns with the runway, the orbiter begins a steep descent with the nose angled as much as 19 degrees down from horizontal. This glide slope is seven times steeper than the average commercial airliner landing. During the final approach, the vehicle drops toward the runway 20 times faster than a commercial airliner as its rate of descent and airspeed increase. At less than 2,000 feet above the ground, the commander raises the nose and slows the rate of descent in preparation for touchdown. After landing the vehicle stands on the runway to permit the fumes from poisonous hydrazine that was used as propellant for attitude control to dissipate.

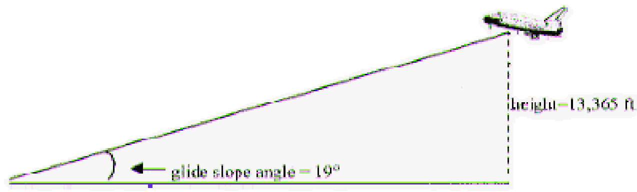


Fig. 53: Landing of the Orbiter Atlantis

The Energy of the Orbiter on Entering the Atmosphere at About 121 Km

The Orbiter enters the atmosphere with a velocity of about 7800 m/s at an altitude of about 121 km. The mass of the Orbiter is about 70,000 kg.

The energy of the Orbiter is due two its motion and its gravitational potential energy. First we will find the kinetic energy of the Orbiter and then its potential energy at an altitude of 121 km.

The kinetic energy is given by

$$\mathbf{E_k = \frac{1}{2}\, m\, v^2} \text{ and } \mathbf{E_p = mgh}$$

(Since the value of g at 121 km altitude is about 96% of its value (9.81 m/s^2) on earth, we assume that the average is about 9.6 m/s^2).

$$\mathbf{E_k = \frac{1}{2} \times 70{,}000 \times (7800)^2 = 2.13 \times 10^{12}\ J.}$$

$$\mathbf{E_p = 70{,}000 \times 9.6 \times 121{,}000 = 8.13 \times 10^{10}\ J.}$$

The total energy of the Orbiter in coming to a full stop on the surface of the earth is

$$\mathbf{2.21 \times {}^{12}\ J}$$

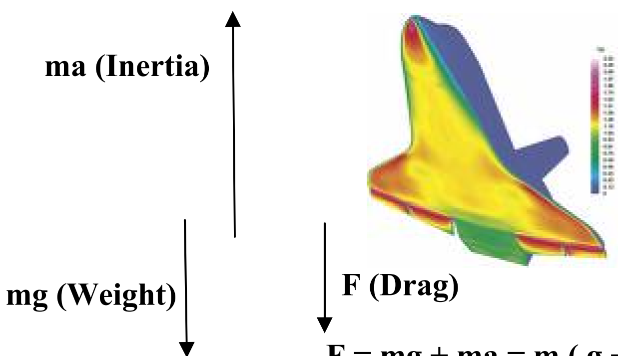
It seems that only about 4% of the total energy is gravitational potential energy. But how can we get a sense of how much this energy is? One way is to ask the question "how many gallons of gasoline contain that much chemical potential energy?"

Gasoline contains about 4.4 x 10^7 Joules per kilogram of combustion energy. So we divide 2.21 x ^{12} J. by 4.4 x 10^7 to get an answer of about 50,227 kg of gasoline. This is equal to about 64,000 litres of gasoline. An average car tank holds about 40 litres, so this amounts to about 1600 tans full of gasoline. Assuming that you can travel about 10 km/l this amount of gasoline could take you a distance of 640,000 km, or 16 times around the earth!

Checking the Region of Highest Deceleration

Using the acceleration- time, the velocity-time, as well as the altitude-time graphs above shows that the largest deceleration occurs between about 47km and 30 km, between about 18min. and 22 min. prior to touchdown. We will pick the deceleration at 22 min. prior to touchdown, at an altitude of about 35 km. The deceleration is about 7.5 m/ s^2. and the atmospheric density about 0.0082 kg/m^3 .We will first calculate the force necessary to slow down the Orbiter at this rate. The only force that is available to accomplish this is the drag force provided by the atmosphere as the Orbiter moves through the air at a high speed.

The drag force, however, must provide two forces: the weight of the Orbiter as well as the inertial force ma. So we have:



F = mg + ma = m (g + a)

Fig. 54: Net Force on the Returning Space Shuttle

To calculate the force F necessary to maintain the deceleration of about 7.5 m/ s^2:

The mass of the Orbiter is about 70,000 kg and we will take the effective area exposed to the atmosphere at this point as about 100 m^2. (The total area of the lower surface of the Orbiter is about 300 m^2, but the Orbiter is at an angle of about 40º.

W = mg = 70,000 x 10 = 700,000 N, A = 100 m^2

F = W + ma = 700,000 + 70,000 x 7.5 = **1.22 x 10^6 N.**

This is the retarding force necessary to produce a deceleration of 7.5 m/ s^2. This force must be provided by the drag of the atmosphere. In calculating the drag we will use the following data:

Effective area of Orbiter: 100 m^2.

Drag coefficient: 1.0

Density of the atmosphere: 0.0082 kg/m^3

Velocity of the Orbiter: 1.80 km/s (1800 m/s)

Using the equation for drag, **½ A ρ v^2 C_D** we have:

½ x100x 0.0082 x $(1800)^2$ x 1.0 = **1.3 x 10^6 N.**

This is a very satisfactory result for a first-order calculation. What is the weight of an astronaut whose mass is 80 kg at this time?

Questions and Problems

Part A

1. Estimate the energy it took to get the Orbiter into a stable orbit at 125 km.

   Follow the pattern shown above in calculating the energy required to safely land the Orbiter on earth. Follow the trajectory description of the ascent above and calculate the energy required for the two stages. Show that the total energy is about 4.2×10^{12} J, or about 2 times the energy required to slow the Orbiter from about 8 km/s to rest.

2. The initial orbiter landing approach is at a glide slope of 19 degrees (See Fig. above). This is six times steeper than the 3-degree glide slope of a typical commercial jet airliner as it approaches landing. Just before the orbiter touches down, flare or pull-up maneuvers are required to bring it into its final landing glide slope of l.5 degrees. At touchdown -- nominally about 2,500 ft. beyond the runway threshold -- the Orbiter is traveling at a speed ranging from 213 to 226 mph. The drag chute opens (d =10m) and at approximately 56 km per hour, the chute completes its work and disconnects from the Orbiter. The Orbiter then rolls to a stop using its brakes.

   Assume that the Orbiter lands at 226 mph (362 kph, or 101 m/s) when the drag chute opens and disconnects when the Orbiter is moving at 56 km per hour.

   a. What is the descent rate in feet per second and in m/s? (Notice that in Fig. the distance in feet is incorrect. it should be about 40,000ft)

   b. Find the drag force produced by the parachute, called a drag chute, that has a diameter of 12m.

   c. Calculate the initial deceleration on the Orbiter.

   d. Estimate the time and the distance traveled while the drag chute is in operation. Remember that the drag force varies with the speed.

3. The following is taken from a NASA publication (see IL, above)

   The orbiter's environmental control and life-support system purifies the cabin air, adds fresh oxygen, keeps the pressure at sea level, heats and cools the air, and provides drinking and wash water. The system also includes lavatory facilities. The cabin is pressurized to sea level (14.7 psi) with 21 percent oxygen and 79 percent nitrogen, comparable to earth's atmosphere. The air is circulated through lithium hydroxide/charcoal canisters which remove carbon dioxide. The canisters are changed on a regular basis. Excess heat from the cabin and flight-deck electronics is collected by a circulating coolant water system and transferred to radiator panels on the payload bay doors where it is dissipated. The fuel cells produce about seven pounds of water each hour. It is stored in tanks, and excess water is dumped overboard. The lavatory unit collects and processes body waste, and also collects wash water from the personal hygiene station. The lavatory unit, located in the mid deck area, operates much like those on commercial airlines but is designed for a weightless space environment.

   Assignment: Two students should find out more detail about the above and make a class presentation.

4. The thermal protection system is designed to limit the temperature of the orbiter's aluminum and graphite epoxy structures to about 350 degrees (F) during reentry. There are four types of materials used to protect the orbiter. Reinforced carbon-carbon is a composite of a layer of graphite cloth contained in a carbon matrix. It is used on the nose cap and wing leading edges where temperatures exceed 2,300 degrees (F). High-temperature reusable surface insulation consists of about 20,000 tiles located mainly on the lower surfaces of the vehicle. The tiles are about six inches square and made of a low-density silica fiber insulator bonded to the surface in areas where temperatures reach up to 1,300 degrees (F). Low-temperature reusable surface insulation also consists of tiles. There are about 7000 of these on the upper wing and fuselage sides where temperatures range from 700 to 1,200 degrees (F). Flexible reusable surface insulation (coated Nomex felt) is sheet-type material applied directly to the payload bay doors, sides of the fuselage and upper wing areas where heat does not exceed 700 degrees (F).

   <u>Assignment</u>: Two students should find out more detail about the above and make a class presentation

Questions and Problems

Part B

1. Find the approximate atmospheric drag on the Shuttle for the following (first study the terrestrial problem of drag in LCP 3):
   a. At the beginning of the descent ?
   b. At 70 km: maximum heating effect?
   c. At 600 m, just before landing?.
2. What is the average deceleration of the Shuttle between entering the atmosphere at L-0.30 and the end of the communication blackout at L-0:12?
3. Estimate the maximum deceleration experienced by the Shuttle and the crew. By what percentage will the weight of an astronaut increase during this time?
4. Estimate the drag force at 70 km and compare that to the actual average breaking force that the Shuttle experienced. What percentage of this is the drag force? Comment.
5. Discuss the energy budget of the re-entry of the Orbiter according to the following:
   a. The kinetic energy when entering the atmosphere.
   b. The gravitational potential energy when entering the atmosphere
   c. The energy lost due to atmospheric drag
   d. Estimate the change of temperature of the Orbiter, assuming a mass of about 70,000 kg and an average specific heat of 0.2

The Orbiter Sub-systems

(Small research projects for students)

A good brief description of the science of technology of the environment in the Orbiter is given in IL 42and IL 43 below. Read this carefully before answering the questions below.

IL 42 ***

IL 43 ***

STUDENT EXPERIMENTS IN THE SHUTTLE

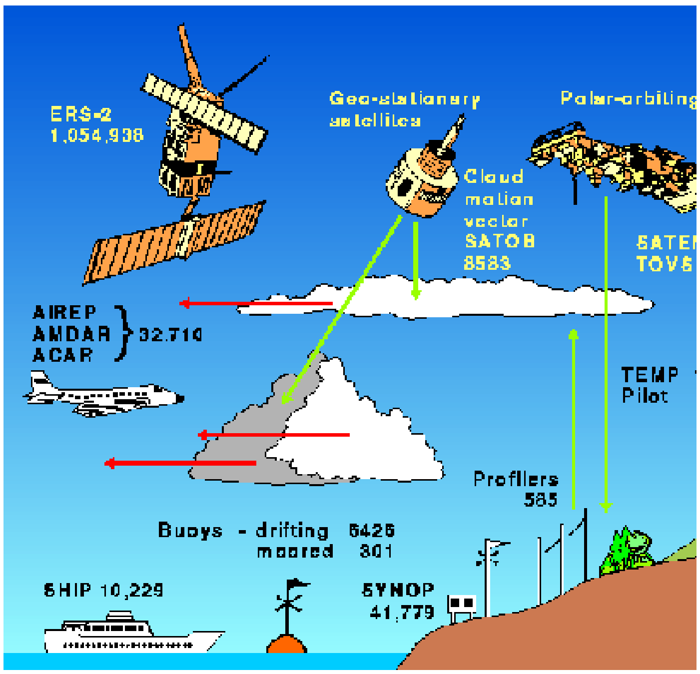
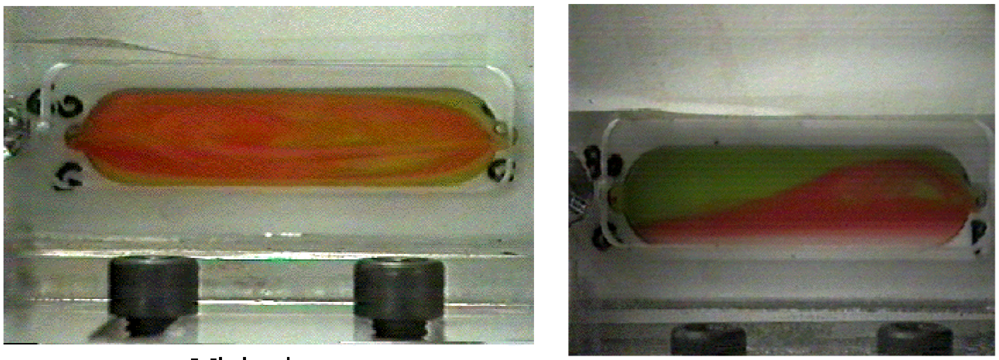


Fig. 55: Experiments in the Orbiter

Fig. 56: Relaxing in the Orbiter



Mixing in μg Mixing in 1-g

Fig. 57: A Systematic Study of Fluids under Microgravity Conditions

The two video clip are from flow visualization experiments. Researchers performed one in reduced gravity and the other under the same conditions in 1g on Earth. You will see a mixing chamber loaded with green solution which is less dense than the red fluid which will be injected into the chamber. The fluid colors result from a special dye which shows the pH of each solution. The red solution is diluted household vinegar. The green solution is a water-based solution. Different colors show the various pHs of solutions as they combine in the chamber and indicate the patterns of fluid motion.

The Canadian astronaut Bjarni Tryggvason was on board the space shuttle Discovery, in August of 1997. He flew as a payload specialist aboard Space Shuttle *Discovery* on Mission STS-85. His role included to perform fluid science experiments designed to examine sensitivity to spacecraft vibrations. During that flight he also carried out several student experiments. In one of them he wanted to measure to the drag effect of the atmosphere and in another he measured the mass of unknown metallic sphere. See IL for details of this mission.

IL 44 ** Biography of Bjarni Tryggvason

IL 45 *** Results of experiments in microgravity

IL 46 ** Results of experiments in microgravity

The first experiment can actually be used to calculate, first, the drag force, secondly, the energy loss for each orbit and the corresponding drop in altitude, and finally the molecular density at the height of 300 km.

The second experiment can be used to measure the mass of an object in microgravity by its inertial properties.

Fig. 58: NRC’s original Canadian Space Team
Back row (l-r): Bjarni Tryggvason, Robert Thirsk, Roberta Bondar, Steve MacLean.
Front row (l-r): Ken Money and Marc Garneau.

IL 47 *** Student experiments in the Shuttle

Preliminary Calculations: Drag Measurement on the Orbiter

We often hear that there space is empty. In reality, at the altitudes of the orbits for the shuttle there are of the order of 10 billion molecules per cubic centimetre. While that is a large number density of molecules, the physical density is small compared to the number here at the surface of the earth. The pressure and density at the shuttle’s orbital altitude therefore is extremely small.

At an altitude of 300 km the density is about $5x10^{-14}$ g/cm^3, as compared to about $1.3x10^{-3}$ g/cm^3 on the surface of the earth. Therefore, the density on earth is one hundred billion times as high! However, with the very high speed of the shuttle, there still is a frictional effect. This friction causes the shuttle to loose energy, which will appear as a combination of slowing down and dropping in altitude. The decrease of speed is a deceleration which can be observed by carefully measuring the motion of objects inside the shuttle. The deceleration is very small, and any object used to observe this effect will need to be protected form the air movement within the shuttle cabin.

Based on our discussion earlier of the drag force in the Shuttle ascending and on the Orbiter descending we can estimate the drag force on the Shuttle in the low density atmosphere of an orbit. The drag force on the Shuttle should be given by: $\mathbf{F_D = -\frac{1}{2}\,\Delta\, A\, C_D\, v^2}$. Notice that, unlike for the terrestrial case of a vehicle on a level highway, this is the only force that is acting on the Shuttle in the direction of the motion.

The density Δ of the atmosphere at 300 km is about $5x10^{-11}$ kg / m^3 . The area of exposure for the Shuttle is abou t 10 m^2 and the orbital velocity of the Shuttle is about $8x10^3$ m/s. We will take the drag coefficient to be about 1.

Therefore, the drag force acting on the Shuttle at this height and velocity should be about

$$- ½ \times 5x10^{-11} \times 10 \times 1 \times (8x10^3)^2 \text{ N, or about } 0.02 \text{ N.}$$

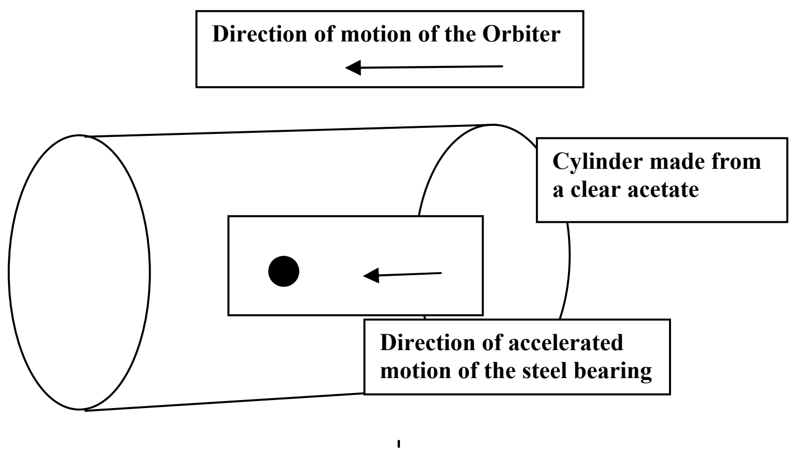


Fig. 59: The Small Metallic Sphere Is Moving Slowly Toward The Front Of The Orbiter

Experiment: Determining the Deceleration of the Orbiter

In this experiment, a clear acetate was rolled into a cylindrical form. A steel spherical bearing was then placed inside, and both ends taped closed. By observing the position of the bearing over time, the acceleration of the shuttle can be determined. This can be used to calculate:

- The drag force acting on the shuttle;
- The decay in the orbital altitude per orbit;
- The balance between the work done and the reduction in shuttle energy;
- A small magnet can be used to reposition the bearing to one end of the tube to start or repeat the experiment.

Bjarni Tryggvason found that the metallic <u>sphere drifted about 5 cm in 6 minutes toward the front of the Shuttle.</u>

Problems and Questions Based on the Experiment

1. Using this simple result we can calculate:
   a. The microgravity inside the Shuttle
   b. The average drag force on the Shuttle
   c. The energy loss in one revolution
   d. The corresponding loss in height of the Shuttle
   e. The density of the atmosphere at that height.

Note: For these approximate calculations use:

M (orbiter) $= 1\text{x}10^{5}$ kg

R (earth) $= 6.4\text{x}10^{6\ \text{kg}}$

R (orbit) $= 6.7\text{x}10^{6}$ kg

M (earth) $= 6\text{x}10^{24}$ kg

G $= 6.7\text{x}10^{-11}\ \text{N.m}^2 / \text{kg}^2$

Problems and Questions

1. Show that:
   a. The acceleration of the metallic sphere is about $7\text{x}10^{-7}\ \text{m/s}^2$ .That means that we have truly "microgravity" conditions in the orbiting Shuttle, because this is almost 10^{-6} g's!
   b. The average drag force therefore must be about $7\text{x}10^{-2}$ N.
   c. Since the Shuttle travels around the earth the total energy involved in slowing the Shuttle down is $3.2\text{x}10^{6}$ J.
   d. The loss in height is about 10 m for each revolution.
   e. The density of the atmosphere is about $2\text{x}10^{-10}\ \text{kg/m}^3$, or $2\text{x}10^{-13}\ \text{g/cm}^3$.
2. If the Shuttle stays in orbit for 10 days estimate the height loss it would experience, if there were no compensation made.
3. Estimate the number of "particles" per cubic centimetres at this height if the density of the atmosphere on the surface is about $1.3\text{x}10^{-3}\ \text{g/cm}^3$ and the number of particles about $3\text{x}10^{19}$ per cm^3. (Note: There are $6\text{x}10^{23}$ molecules in 22.4 l of a gas at STP). You should find that there are still about 10^{9} particles in each cubic centimetre at this height.
4. An interesting question is the following: How large a sphere, made of material whose density is that of the earth (about 5500 kg/m^3), would produce a gravity at its surface equal to the microgravity found on the Shuttle? You should be able to show that a sphere of less than 1 m radius would produce that gravitational effect. That means if you placed a small object like the metallic bearing used for the experiment on the Shuttle about 5 cm above the sphere it would take 6 minutes for it to "fall" to the surface of the small sphere.
5. Finally, compare the prediction we made about the expected drag force on the Shuttle and comment.

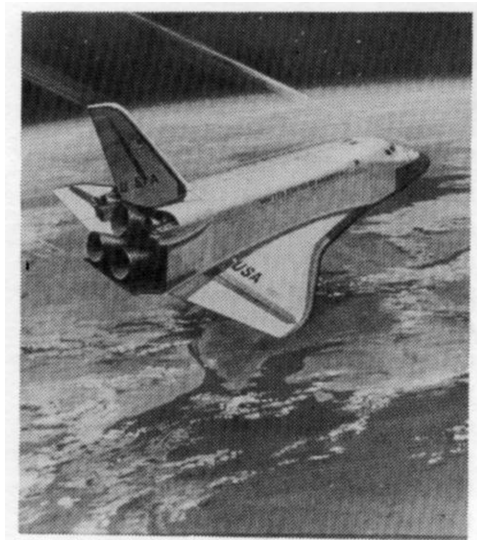


Fig. 60: The Atmospheric Drag on the Orbiter

The Aurora, the Stars and Light from the Earth and from Space

We can see the Aurora at night, and we can see stars at night. What happens to these during the day. Why can we not see these things during the day? What can we see from the Shuttle during daytime and night time?

How far can you see a candle on a dark night in the country where light pollution is low? Surprisingly, the answer is: About 50 km!

How far can an astronaut see from the Orbiter (from about 200km above the Earth surface)? This problem can be solved by drawing a large circle (earth), say 1 m in diameter, and connect the tangents to the scaled height.

Problems and Questions

1. You can calculate the number of photons that must travel through your eyes every second if you can see a candle on a dark night? Assume that the area of your eye is about 1 cm^2.

   First, assume that the light energy output of a candle is about 1 J/s. Then use the inverse square law and Planck's relationship $E = hf$, where h is Planck's constant and f is the frequency of light (use 5000 nanometres. Why?) to solve the problem.

2. You may want to know the limit of the your perception the following way: Experiments have shown that the eye can react to about 10 photons per second. So you should be able to perceive a candle light in pitch dark from much farther than our assumed 10 km. What distance would it be? Comment.

3. Study Fig. 62 carefully before calculating the distance you could see from the Shuttle from a height of about 300 km. Could you see from the Atlantic coast to the Pacific coast? Comment.

Fig. 61: How far can you see a candle in the dark?

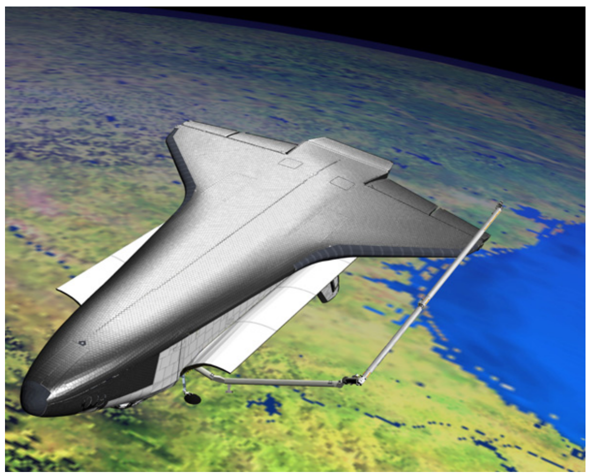


Fig. 62: How Far Can You See From The Orbiter At An Altitude Of 300 Km?

Using a Balance to Measure Mass in Space

Bjarni Tryggvason also demonstrated how it is possible to determine the mass of an unknown object in the microgravity environment. We have already calculated the acceleration of an object on the Shuttle and found it to be about $1x10^{-7}$ m/s^2. We also know that the gravity of that magnitude would be produced by an earth-like sphere that has a radius of about 1 m

If we placed a 1 kg mass on a super balance that rested on the surface we would read about $1x10^{-6}$ N. Clearly, we could not measure the weight of an object in the Shuttle by using a Newton balance. However, we can compare two masses, one known and the other unknown by taking advantage of the fact that gravitational mass and inertial mass are equal!

In this experiment Bjarni Tryggvasen demonstrated how the mass of an unknown object could be determined in the microgravity environment of the Orbiter. An unknown mass is attached to one end of a metal rod. A known mass is attached to the other. The rod is lifted (accelerated) "upwards" by supporting it at a pivot point. The location of the pivot point is adjusted until a point of balance is found so that the system may be accelerated without rotation. The unknown mass can be calculated by measuring the location of the pivot relative to the two masses.

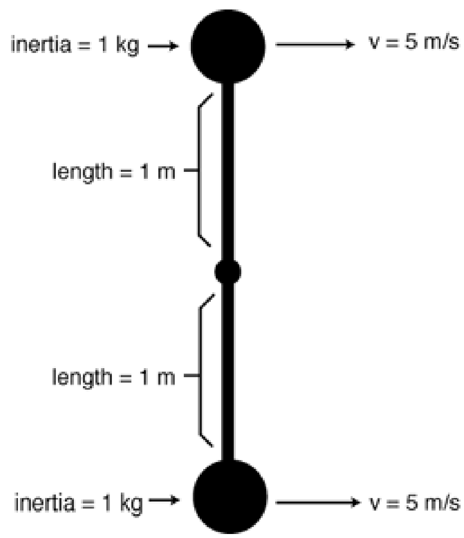


Fig 63: Comparing Two Masses Based In Inertia

Of course, you can also find the unknown mass of the object by maintaining the pivot at the centre of the rod and changing the known mass until balance is obtained when the system is accelerated.

When this happens the unknown mass must be equal to the known mass.

Question

1. How does this simple experiment show the difference between mass and weight?
2. Go back to LCP 1 and review the section on Einstein's General Theory of Gravity, especially his *Equivalence Principle*. Use this principle to explain the method of measuring the mass of an unknown object.
3. How do we measure mass on the earth? Can we devise a means for measuring mass in free fall that uses the same principle as the measurement of mass on earth?

The Maple Leaf Challenge

This is a special challenge to all students. In the Maple Leaf Challenge, the lights of several cities across Canada will be turned off in the pattern of a maple leaf that the crew of the Discovery can see it from space. Your challenge is to determine how large this maple leaf has to be in order that the crew will be able to see it. The Shuttle will be orbiting at an altitude of 160 nautical miles, and will pass over Canada on many of its orbits, such that at almost all large cities in Canada will be within the possible field of view of the Shuttle.

How large does the maple leaf have to be for the crew to see it?

The following need to be considered:

The height of the Orbiter

The resolution of our eyes

The viewing angle

The contrast levels between areas with the lights off and those with the lights on

What contrast would be required to see an object on the ground during daytime?

Young students could understand the following argument (with visuals provided, of course): Assume that you can distinguish between two lines (of bright and dark origin) that are 0.1mm in width and 1m away. The ratio of .1mm to 1m is 1/10000 (sin of the angle). It follows, by similar triangles, that one should be able to see an object about 10m “long” from a height of 100 km. Therefore, from the height of 200 Km (roughly the height of the Shuttle you should be able to distinguish between objects 20m “long”.

You could, for example, see the CN tower.

Fig. 64: View of the Earth from the Orbiter.

Fig. 65: View of the Great Wall of China from Space