INSTRUCTIONS:
- Complete solutions are to be written in the booklet provided.
- Begin each question on a separate page and clearly indicate the question number.
- Return the exam manuscript and all extra pages at the end of the exam.

*** FIVE OF THE EIGHT QUESTIONS ARE TO BE ANSWERED ***
THE EXAM IS THREE HOURS

ALLOWED MATERIAL:
- ONE textbook (hardcopy) of the student’s choice.
- Calculators with empty memory and no pre-stored programs.
- ONE 8-1/2” x 11” formula sheet (writing on both sides allowed).
- Cell phones or other electronic devices are not permitted and must be turned off.

Suggested study textbook and preparation for the examination:
- Topics covered in the examination may include:
  - Electrostatics and Magnetostatics
  - Maxwell’s equations and Electromagnetic theory
  - Wave propagation and Waveguides
  - Transmission line theory and Microwave circuits
  - Radiation and Antennas

STUDENT NUMBER: _______________________________
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*** Circle which 5 questions you want to be marked on the table below.

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Total of 5 questions 100
1. A very long straight wire carrying a sinusoidal with time current, \( i(t) = I \cos \omega t \) [A], is coplanar with a rectangular loop of side lengths \( a \) and \( b \), as shown in the figure below. The two sides of length \( b \) are parallel to the straight wire. The permeability is everywhere \( \mu_0 \).
   
a) Apply the integral form of Faraday’s law to find the magnitude and direction of the electromotive force induced in the loop.
   
b) Use the point form of the Faraday’s law to determine the electric field intensity induced at any point in space by the straight wire current. Calculate the electromotive force induced in the loop and compare with the result obtained in a), (employ circular cylindrical coordinates).
   
c) Find the mutual inductance between the straight wire and the loop. Determine the electromotive force induced in the loop from the mutual inductance.

![Figure: Problem 1.](image)

2. What are the relations among the constants required for each of the following to be a solution of the three-dimensional Helmholtz equation?

   a) \( E_x = C \sin k_x x \sin k_y y \sin k_z z \)

   b) \( E_x = C \sinh K_x x \sin k_y y \sin k_z z \)

   c) \( E_x = C \sinh K_x x \sinh K_y y \sinh K_z z \)
3. The field at a large distance from a dipole radiator has the form, in spherical coordinates as given below. Find the average power radiated through a large sphere of radius \( r \).

\[
E_\theta = \frac{\mu}{\sqrt{\epsilon}} H_\phi = \frac{A}{r} e^{-jkr \sin \theta}
\]

Note: \( \int \sin^3 x \, dx = -\frac{1}{3} \cos x (\sin^2 x + 2) \)

4. Space is filled by two dielectrics, \( \epsilon_1 \) filling half-space \( x > 0 \) and \( \epsilon_2 \) filling half-space \( x < 0 \). Determine whether or not there can exist a uniform plane wave with \( E_x \) and \( H_y \) only and no variations with \( x \) or \( y \), propagating in the \( z \) direction in this composite dielectric. The propagation factor may be \( e^{-jkz} \) with any value if \( k \). Note that the wave, if it exists, must satisfy the wave equation in each region and the continuity conditions at the plane between the two regions.

Figure: Problem 4.

5. Calculate the reflection coefficient and percent of incident energy reflected when a uniform plane wave is normally incident on a plexiglas radome (dielectric window) of thickness \( \frac{3}{8} \) in., relative permittivity \( \epsilon_r = 2.8 \), with free space on both sides. Frequency corresponds to free-space wavelength of 20 cm. Repeat for \( \lambda_0 = 10 \) cm and 3 cm.
6. The circuit shown below includes; a 3-port device with S-parameters as specified, a lossless transmission line, two loads and a source. Port 2 is connected to the lossless transmission line characterized by \( \varepsilon_{\text{reff}} = 9 \) and \( Z_0 = 50 \Omega \) and is of length 1.5 m. The transmission line is terminated with a 350\( \Omega \) load. Port 3 is terminated with a \( Z_0 = 50 \Omega \) load. Assume a reference impedance \( Z_0 = 50 \Omega \) and an operating frequency of \( f = 50 \text{ MHz} \).

a) If a source with \( V_s = 8.0 \angle 0^\circ \text{ V} \) and impedance \( Z_s = 50 \Omega \) is connected to Port 1, determine the \textbf{TOTAL voltages}, \( V_1 \), \( V_2 \) and \( V_3 \) at the 3 ports.

\[ V_1 = ?, \quad V_2 = ?, \quad V_3 = ? \]

b) Show whether or not the 3-port device in the circuit (indicated by the S matrix) below is lossless.

![Diagram of the circuit](image)

Figure: Problem 6.
7. Assume a two-antenna system consisting of two infinitesimal electric dipole antennas as shown below. The currents of these two antennas are \( I_0 e^{+j\pi/4} \) and \( I_0 e^{-j\pi/4} \).

a) Find all possible separation distances, \( d \), between these two antennas so that there would be a null in the radiation pattern of this antenna system along \(+z\) direction.

b) Does the current distribution on the antenna matter in this problem - explain?

![Figure: Problem 7.](image)

8. Assume an aperture antenna of size \( a \times b \) that is mounted on an infinitely-large perfect electric conductor (PEC) plane as shown below. In addition, assume that the electric field distribution on the aperture is uniform, and is given by \( E = E_0 \hat{y} \). Find the far-field electric field of this aperture antenna.

![Figure: Problem 8.](image)