PhD Candidacy Examination
Control Systems
Sample Questions

Chapter 1

Analysis and Design

Problem 1.1

Determine the phase margin and gain margin for the system in which

$$GH(j\omega) = \frac{1}{(j\omega + 1)^3}$$

■

Problem 1.2

Determine the bandwidth for the system with transfer function:

$$\frac{1}{s+1}$$

■

Problem 1.3

Determine the resonance peak M_p and the resonant frequency ω_p for the system whose transfer function is:

$$\frac{5}{s^2 + 2s + 5}$$

■

Problem 1.4

Sketch the Nyquist Stability Plot of

$$GH(s) = \frac{1}{s^3(s+1)}$$

■

Problem 1.5

Sketch the Nyquist Stability Plot of

$$GH(s) = \frac{K}{s(s+p_1)(s+p_2)}$$

■

Problem 1.6

Sketch the Nyquist Stability Plot of

$$GH(s) = \frac{K}{s^2(s+p_1)(s+p_2)}$$

■

Problem 1.7

Sketch the Nyquist Stability Plot of

$$GH(s) = \frac{s+z_1}{s^2(s+p_1)(s+p_2)}$$

■

Problem 1.8

Design a compensator which yields a phase margin of approximately 45^0 for the system defined by

$$GH(s) = \frac{84}{s(s+2)(s+6)}$$

■

Problem 1.9

Design a compensator which yields a phase margin of approximately 40^0 and a velocity constant $K_v = 40$ for the system defined by

$$GH(s) = \frac{4 \times 10^5}{s(s+20)(s+100)}$$

■

Problem 1.10

What kind of compensation can be used to yield a maximum overshoot of 20% for the system defined by

$$GH(s) = \frac{4 \times 10^4}{s^2(s+100)}$$

■

Chapter 2

Root Locus Design

Problem 2.1

Sketch the branches of the root-locus for the transfer function

$$GH = \frac{K(s+2)}{(s+1)(s+3+j)(s+3-j)} \qquad K > 0$$

■

Problem 2.2

Construct the root-locus for $K > 0$ and $K < 0$ for the transfer function

$$GH = \frac{K}{s(s+1)(s+3)(s+4)}$$

■

Problem 2.3

Find the departure angle of the root-locus from the pole at $s = -10 + j10$ for

$$GH = \frac{K(s+8)}{(s+14)(s+10+j10)(s+10-j10)} \qquad K > 0$$

■

Problem 2.4

Determine an appropriate compensator $G_1(z)$ for the discrete time unity feedback system with

$$G_2(z) = \frac{3(z+1)(z+1/3)}{8z(z+0.5)}$$

■

Problem 2.5

Design a continuous unity feedback system with the plant G=K/(s+1)(s+3) and the following spescifications: (a) Overshoot of less than 20%, (b) $K_p \geq 4$, (c) 10 to 90 % rise time less than 1s. ■

Problem 2.6

Determine the overshoot and rise time of the system with transfer function

$$\frac{C}{R} = \frac{1}{(s+1)(s^2+s+1)}$$

■

Problem 2.7

Determine a suitable compensator for the system with the plant transfer function

$$G(s) = \frac{1}{s(s+1)(s+4)}$$

to satisfy the following specifications: (1) overshoot less than 20%, (2) 10 to 90 % rise time $\leq$ 1s, and (3) gain margin $\geq$ 5. ■

Chapter 3

Frequency Response Design

Problem 3.1

Construct Bode plots for the frequency response function,

$$GH(j\omega) = \frac{2}{j\omega(1 + j\omega/2)(1 + j\omega/5)}$$

■

Problem 3.2

Design a compensator for the uncompensated continuous-time system whose open-loop transfer function is

$$GH = \frac{24}{s(s+2)(s+6)} \qquad H = 1$$

to meet the following performance specifications:

1. when the input is a ramp with slope (velocity) 2π rad/s, the steady state position error must be less than or equal to $\pi/10$ radians.
2. $\phi_{PM} = 45^0 \pm 5^0$.
3. gain cross over frequency to be greater than or equal to 1 rad/s.

■

Problem 3.3

Design a compensator for the unity feedback discrete-time system with opem-loop transfer function

$$G(z) = GH(z) = \frac{3}{8}\frac{(z+1)(z+1/3)}{z(z+1)}$$

and sampling period of $T = 0.1$ s to meet the following performance specifications:

1. The steady state error must be less than or equal to 0.02 for a unit ramp input.
2. $\phi_{PM} \geq 30^0$.
3. gain cross over frequency toto satisfy $\omega T \geq 1$.

■

Problem 3.4

The closed loop transfer function between the input $R(s)$ and the output $C(s)$ of a control system is given by:

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 2s + 3}$$

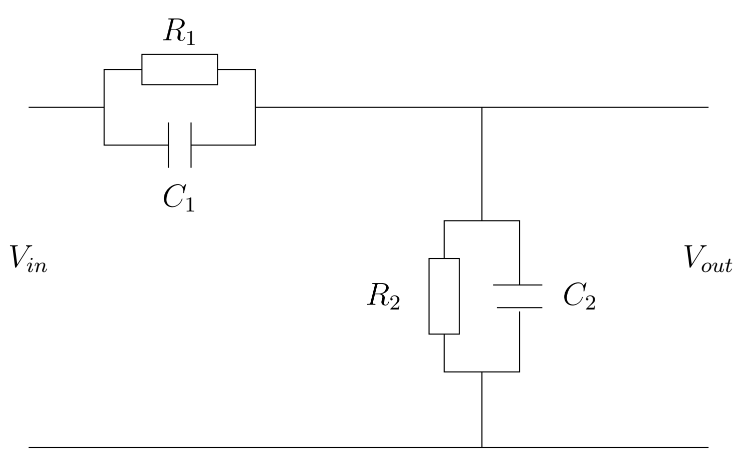
Determine the percentage overshoot and the time to peak for a unit step input. ■

Chapter 4

State Space Design

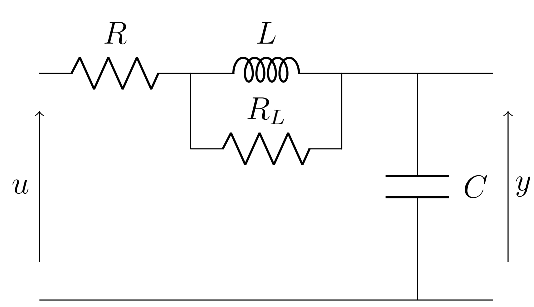
Problem 4.1

Obtain a state-space representation for the circuit shown below. ■



Problem 4.2

Derive a state space model for the circuit shown below.



The values of the circuit parameters are as given below:
$R = 2\Omega$, $R_L = 200\Omega$, $L = 0.05H$, $C = 80\mu F$
Determine the eigenvalues of this system.

■

Problem 4.3

Compute e^{At} for

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$$

■

Problem 4.4

Test the controllability of the following state space system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2.5 & -1.5 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

■

Problem 4.5

Test the observability of the following state space system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

■

Problem 4.6

Consider the system defined by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 3 & -4 & 5 \\ -6 & 7 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 2 & -3 \\ 4 & -5 \end{bmatrix} u$$

Is the system completely state controllable? ■

Problem 4.7

Obtain the response y(t) of the following system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u$$

$$\begin{bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

■

Problem 4.8

Consider the system defined by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1.0 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The initial values of the states are $x_1 = 0$ and $x_2 = 0$. Derive the state transition matrix $\Phi(t)$ and hence obtain the time response $y(t)$.

■