1.) You invest \$1,000 at the end of each year for 5 years at an annual effective rate of 9% and reinvest the interest at an annual effective rate of 9%. At the end of 5 years, your investment has a value of X.

Your friend invests \$1,000 at the beginning of each year for 5 years, but at an annual effective rate of 10% and reinvests the interest at an annual effective rate of 8%. At the end of 5 years, your friend's investment is equal to Y.

Calculate Y – X.

2.) You borrow X and repay the principal by making 10 annual payments at the end of each year into a sinking fund which earns an annual effective rate of 8%. The interest earned on the sinking fund in the third year is \$85.57.

Calculate X.

3.) An investor deposits 50 in an investment account on January 1. The following summarizes the activity in the account during the year:

Date	Value Immediately Before Deposit	Deposit
March 15	40	20
June 1	80	80
October 1	175	75

On June 30, the value of the account is 157.50. On December 31, the value of the account is X. Using the time-weighted method, the equivalent annual effective yield during the first 6 months is equal to the (time-weighted) annual effective yield during the entire 1-year period.

Calculate X.

4.) A loan is being repaid with 25 annual payments of 300 each. With the 10th payment, the borrower pays an extra 1000, and then repays the balance over 10 years with a revised annual payment. The effective rate of interest is 8%.

Calculate the amount of the revised annual payment.

5.) The following table shows the annual effective interest rates being credited by an investment account, by calendar year of investment. The investment year method is applicable for the first 3 years, after which a portfolio rate is used:

Calendar Year of Investment	Investment Year Rates i_1	i_2	i_3	Calendar Year of Portfolio Rate	Portfolio Rate
1990	10%	10%	t%	1993	8%
1991	12%	5%	10%	1994	$(t-1)$%
1992	8%	$(t-2)$%	12%	1995	6%
1993	9%	11%	6%	1996	9%
1994	7%	7%	10%	1997	10%

An investment of 100 is made at the beginning of years 1991, 1992, and 1993. The total amount of interest credited by the fund during the year 1995 is equal to 22.81.

Calculate t.

6.) Two $1,000 face amount bonds are purchased. The 2n-year bond costs $250 more than the n-year bond.

Each has 13% annual coupons and each is purchased to yield 6.5% annual effective.

Calculate the price of the n-year bond.

7.) You take out a 10 year loan of $4,600. Interest on the loan is charged at an annual rate of interest of 9%.

You repay the loan by making 10 annual payments at the end of each year consisting of interest on the loan and a sinking fund deposit. The total payment each year is $743.

The amount of the sinking fund will exactly pay off the loan at the end of 10 years.

During the first 8 years of the loan, the sinking fund annual effective rate of return is 7.75%. After year 8, the sinking fund's annual rate of return is i.

Calculate i.

8.) An annuity-due is purchased at a price of \$2,500. The annuity makes annual payments for 5 years. The first payment is \$975 and each subsequent payment decreases by \$150.

You are also given:

- The payments are reinvested in a fund which earns an annual effective rate of 6%.
- Interest payments from the fund are received every year and reinvested at an effective annual rate of z%.
- z is the equivalent effective annual rate earned on a \$1,000 face amount, 11-week T-bill with a discount yield of 4.321%.
- The overall effective annual yield on this investment over the 5-year period is i.

Calculate i.

9.) A loan of \$10,000 is amortized by equal annual payments for 30 years at an effective annual interest rate of 5%. Determine the year in which the interest portion of the payment is most nearly equal to one-third of the payment.

10.) Two bonds are purchased for the same price to yield 5%. Bond X has 4% annual coupons and matures for its face value of \$100. Bond Y has annual coupons of \$3 and matures for \$180. Both bonds mature at the end of n years.

Calculate n.

Test 3 Solutions

(1.) $X = 1000\, s_{\overline{5}|.09} = 5984.71$

$$Y = 5\times 1000 + \left[\overset{P}{\overset{\shortparallel}{1000(.10)\cdot a_{\overline{5}|8\%}}} + \overset{Q}{\overset{\shortparallel}{1000(.10)\cdot \left[\frac{a_{\overline{5}|.08} - 5v^5_{.08}}{.08}\right]}} \right] \times (1.08)^5$$

$$= 5000 + \left[399.271 + 100\left[7.3724\right]\right] \times 1.4693$$

$$= 5000 + 1669.876 = 6669.91$$

$$\Rightarrow Y - X = 6669.91 - 5984.71 = \boxed{685.17}$$

(2.) (S.F. balance at $t=2$) $\times .08 = 85.57$

$SFP = \dfrac{X}{s_{\overline{10}|.08}}$ S.F. Bal at $t=2$ $= \left(\dfrac{X}{s_{\overline{10}|.08}}\right)\cdot s_{\overline{2}|.08}$

$$= \frac{X}{14.487}\cdot 2.08 = .1436X$$

$\Rightarrow (.1436X)\times .08 = 85.57$

$\boxed{X = 7449.61}$

(3) 6-month time weighted yield:

$$= \left(\frac{40}{50}\right)\times\frac{80}{(40+20)}\times\frac{157.50}{(80+80)} - 1 = 1.05 - 1 = .05$$

equivalent effective annual yield $\Rightarrow i = 1.05^2 - 1 = 1.1025 - 1 = .1025$

1 year time weighted yield:

$$= \left(\frac{40}{50}\right)\left(\frac{80}{60}\right)\left(\frac{1175}{160}\right)\left(\frac{X}{175+75}\right) - 1 = .1025 \Rightarrow (.00466)X - 1 = .1025$$

$$\Rightarrow \boxed{X = 236.25}$$

Test 3 Solutions.

(4.) PV of loan $= 300\, a_{\overline{25}|.08} = 3202.43$

PV at $t=10$ $= 300\, a_{\overline{15}|.08} = 2567.84$

... less paydown $\Rightarrow 2567.84 - 1000 = 1567.84$

$\Rightarrow$ new pymt: $1567.84 = X \cdot a_{\overline{10}|.08}$ $\Rightarrow \boxed{X = 233.65}$

(5.) Interest Earned on 1st \$100:

$$100(1.12)(1.05)(1.10) \times (1+t-.01)(.06) = 7.683984 + 7.7616t$$

Interest Earned on 2nd \$100:

$$100(1.08)(1+t-.02)(1.12)(.06) = 7.112448 + 7.2576t$$

Interest Earned on 3rd \$100:

$$100(1.09)(1.11)(.06) = 7.2594$$

$\Rightarrow$ Total Interest $= 22.81 = 22.055832 + 15.0192t$

$$\Rightarrow \quad t = \frac{.754168}{15.0192} = .0502 \text{ or } \boxed{5.02\%}$$

Test 3 Solutions:

(6.) Price of 2n bond = Price of n-yr bond + 250

$$130\, a_{\overline{2n}|} + 1000v^{2n} = 130\, a_{\overline{n}|} + 1000v^{n} + 250$$

$$130\frac{(1-v^{2n})}{.065} + 1000v^{2n} = 130\frac{(1-v^{n})}{.065} + 1000v^{n} + 250$$

$$2000 - 2000v^{2n} + 1000v^{2n} = 2000 - 2000v^{n} + 1000v^{n} + 250$$

$$2000 - 1000v^{2n} = 2250 - 1000v^{n}$$

$\Rightarrow 1000v^{2n} - 1000v^{n} + 250 = 0$ use quadratic w/ $x = v^{n}$

$x = .5 = v^{n}$ $\Rightarrow$ Price of n-year $= 130\frac{(1-.5)}{.065} + 1000(.5) = \boxed{1500}$

(7.) interest payment on loan (SP) $= 4600(.09) = 414$

$\therefore$ SFP $= 743 - 414 = 329$.

AV of Sinking Fund at $t=8$: $329\, s_{\overline{8}|7.75\%} = 3468$

Since AV at $t=10$ must equal 4600, we get that:

$$3468(1+i)^2 + 329(1+i) + 329 = 4600$$
$$3468(1+i)^2 + 329(1+i) - 4271 = 0$$

$\therefore$ let $x = (1+i)$ and use quadratic.

$x = (1+i) = 1.06333$ $\therefore$ $\boxed{i = 6.333\%}$

Test 3 Solutions.

8. $1+z = \left(\frac{\text{face amount}}{\text{price}}\right)^{\wedge \frac{365}{11\times 7}}$; price T-bill $= 1000\left(1-(.04321)\times\frac{11\times 7}{360}\right)$

price = 990.76

$z = \left(\frac{1000}{990.76}\right)^{\wedge \frac{365}{77}} - 1 = 4.5\%$

time	0	1	2	3	4	5
pymt	975	825	675	525	375	0
Σ pymts	975	1800	2475	3000	3375	
int paid (Σ pymts $\times$ .06)		58.5	108	148.5	180	202.5

Total return amount at $t=5$:

$58.5(1.045)^4 + 108(1.045)^3 + 148.5(1.045)^2 + 180(1.045) + 202.5 +$

$3375 = 4120.77$

$\therefore$ (original investment)$\times(1+i)^5 = 4120.77 \Rightarrow (1+i)^5 = \frac{4120.77}{2500}$

$\boxed{i = 10.51\%}$

Test 3 Solutions.

(9.) $10\,000 = X \cdot a_{\overline{30}|5\%}$ $\qquad X = 650.51$

$\frac{1}{3}$ of $X = \dfrac{650.51}{3} = 216.84$

let $I_x = 216.84$ $\qquad I_x = \text{Pymt} - P_x$ $\qquad P_x = P_1 \cdot (1.05)^{x-1}$

$P_1 = 650.51 - 10000(.05) = 150.5144$

$\Rightarrow I_x = 216.84 = 650.51 - (150.51)(1.05)^{x-1}$

$\Rightarrow \dfrac{433.6744}{150.51} = (1.05)^{x-1}$ $\qquad (x-1) = \dfrac{\ln 2.8813}{\ln(1.05)}$

$x = 22.69$, rounding up we get year $\boxed{23}$

(10.) Bond X: Price $= 4 \cdot a_{\overline{n}|5\%} + 100v^n$

Bond Y: Price $= 3 \cdot a_{\overline{n}|5\%} + 180v^n$

$\Rightarrow 4a_{\overline{n}|} + 100v^n = 3a_{\overline{n}|} + 180v^n$

$a_{\overline{n}|} = 80v^n$ $\qquad \dfrac{1-v^n}{.05} = 80v^n$

$\Rightarrow 1 - v^n = 4v^n$ $\qquad 1 = 5v^n$ $\qquad v^n = \dfrac{1}{5}$ $\qquad \Rightarrow \left(\dfrac{1}{1.05}\right)^n = \dfrac{1}{5}$

$n = \dfrac{\ln(1/5)}{\ln\left(\frac{1}{1.05}\right)}$ $\qquad n = 32.99$ $\therefore$ $\boxed{n = 33 \text{ years}}$