

## NUMBER THEORY

### Definitions

1. An integer  $n$  is divisible by an integer  $a$  if there is an integer  $b$  such that  $n = ab$ . (In this case, we also say that  $a$  and  $b$  are divisors, or factors, of  $n$ , or that they divide  $n$ . We also say that  $n$  is a multiple of  $a$  and  $b$ .)
2. An integer  $p > 1$  is a **prime number** if its only divisors are 1 and  $p$ .
3. An integer  $n$  is said to be **even** if it is divisible by 2. If  $n$  is not even, it is said to be **odd**.

### Facts

1. A three digit number  $abc$  can be expressed in the form  $abc = 100a + 10b + c$ ; a four-digit number  $abcd$  can be expressed in the form  $abcd = 1000a + 100b + 10c + d$ .
2. Even and odd integers have the following properties:
  - (a) An even integer can always be expressed in the form  $2k$ , and an odd integer in the form  $2k + 1$ , where  $k$  denotes an integer.
  - (b) Given any two consecutive integers, one is even, the other is odd.
  - (c) even + even = even; odd + odd = even; even + odd = odd
  - (d) even  $\times$  even = even; even  $\times$  odd = even; odd  $\times$  odd = odd
3. Any positive integer has a unique prime factorization; that is, it can be written as a product of prime numbers in one, and, up to the order in which the primes occur, only one way.
4. If  $a$  and  $b$  are both divisible by  $m$ , so also are  $a + b$  and  $a - b$ .
5. When an integer is divided by 3, possible remainders are 0, 1, and 2. It follows that every integer can be expressed in one of the forms  $3k$ ,  $3k + 1$ , and  $3k + 2$  where  $k$  is an integer. Furthermore, given any three consecutive integers, one of them must be divisible by 3. Similar results can be stated for integers other than 3.
6. Some divisibility tests:
  - (a) An integer is divisible by 3 (or 9) if and only if the sum of its digits is divisible by 3 (or 9).
  - (b) An integer is divisible by 5 if and only if the last digit is 0 or 5.
  - (c) An integer with at least two digits is divisible by 4 if and only if its last two digits is divisible by 4.
  - (d) An integer with at least three digits is divisible by 8 if and only if its last three digits is divisible by 8.
  - (e) An integer is divisible by 6 if and only if it is divisible by 2 and by 3.