# Quantitative and Qualitative Evaluation of Geometric Deconvolution of Distortion in Limited-View Computed Tomography

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***Abstract*—Images reconstructed using a set of a few projections spanning a narrow angular range suffer from a systematic geometric distortion due to the point spread function of the reconstruction process. This distortion can be removed by deconvolving a complementary set of projections calculated from the initial reconstruction. Homomorphic deconvolution and inverse filtering techniques were used for this purpose. A second reconstruction is computed from the union of both sets of projections. Although the distortion is removed, the results are noisy due to problems associated with inverse filtering. Two-dimensional (2-D) filtering of the second reconstruction is performed to reduce the noise, while preserving the reduction in geometric distortion. Results obtained using several 2-D filters were compared visually and by point noise content, distortion, and projection error measures. Quantitative measures of geometric distortion removal for test images are also described.**

## INTRODUCTION

IMAGES reconstructed from projections measured over a narrow range of angles suffer from geometric distortion characterized by blurring and elongation along the axes of the projections used. We address the problem of removing this distortion in the context of computed tomography (CT) images from a few ordinary radiographs taken at different angles using overhead X-ray units [1]–[4].

The problem of removing the geometric distortion from the reconstructed image is one of estimating the distorting function and deconvolving it from the reconstruction [3]. If we restrict ourselves to linear reconstruction algorithms (unconstrained multiplicative ART [5], for example), the system point spread function (PSF) is the distorting function we wish to estimate. A further simplification can be made by applying the projection theorem whereby two-dimensional (2-D) operations can be reduced to one-dimensional operations on the projections. The problem is now reduced to that of estimating the PSF component of each of the projections (a form of line spread function), and then deconvolving these projections. These 1-D spread functions can be estimated by deconvolving the projections of a limited-view reconstruction of a test pattern with those acquired directly from the test pattern image. Since the distorted image would have satisfied the projection measurements at all angles for which projection data were given (true of ART-type algorithms), projections at these angles are distortion free. To remove distortions from an arbitrary limited-view reconstruction, a number of projections computed at angles not used in the initial reconstruction are deconvolved with the corresponding spread functions. These computed projections are then used along with the given projections to construct a better estimate of the unknown image (see [3] for details).

The inherent difficulties of deconvolution are noise amplification (the deblurring process is primarily one of high-frequency enhancement) and indeterminacy at the zeros of the spectra of the two signals being deconvolved. These problems can be circumvented by proper construction of the PSF and by the use of constraints based on nonnegativity of the projection data. We suggest that the use of cepstral (homomorphic) processing [6], [7] techniques could make the operation of convolution (or deconvolution) more manageable because spatial-domain convolution becomes an additive operation in the cepstral domain.

## HOMOMORPHIC PROCESSING

Homomorphic systems encompass a wide, extensively investigated [6], [7] class of nonlinear systems capable of transforming a nonlinear operation into a linear one. Systems of the subclass which we will use transform the result of the convolution of two signals in the spatial domain to the sum of their complex cepstra in the cepstral domain.

The complex cepstrum of a data sequence is defined as the inverse $z$ transform of the complex logarithm of the $z$ transform of the data sequence. Computation of the complex cepstrum is complicated by the need for the complex logarithm to be continuous as required by the analyticity condition of the inverse $z$ transform. Therefore, the principal values of the phase of the Fourier spectrum, which would normally form a discontinuous curve, must be corrected by adding appropriate multiples of $2\pi$ to satisfy analyticity. (This is known as phase unwrapping [6]–[9].)

Homomorphic processing will be used in the deconvolution of an unknown sequence from two sequences: a known sequence and a sequence equal to the convolution

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