

A Simulated Comparison of Turnstile and Poisson Photons for X-Ray Imaging

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Abstract

X-rays are used for diagnostic imaging because they produce images with excellent resolution in a cost-effective manner. Due to their carcinogenic potential, however, a trade-off must be made between the amount of x-rays used (dose) and image quality desired. Some noise in x-ray images is attributed to the random nature in which x-rays are emitted from a normal x-ray source. So-called turnstile photon devices have been developed, which release photons in a controlled manner, one at a time. A simple Monte Carlo simulation was created to model specimen radiography, or x-ray of a thin slab. The simulation models two different types of x-ray emitters: regular (Poisson statistics) and turnstile. Projected images were produced for a wide range of doses. These images were compared using mean square error and entropy measures. Results show that image quality would be significantly improved or dose reduced if turnstile photon sources were developed to replace conventional x-ray sources.

Keywords; Turnstile photons, x-ray imaging, dose reduction

1. INTRODUCTION

Getting a clear radiological image is important for the detection of tumors. It has been shown that breast tumor were detected and destroyed before they reached 4 mm diameter, the odds of metastasis would be extremely low, thereby implying that most fatalities due to this cancer could be eliminated [1]. Such resolution is not yet available in current x-ray units at doses acceptable for screening because of the amount of radiation currently needed to get the necessary resolution, and because x-rays are themselves carcinogenic.

Conventional x-ray emitters release photons unpredictably in a manner that can be mathematically modeled as a Poisson distribution in time. This statistical property of emitted photons contributes significantly to the noise in x-ray images. Large numbers of photons must be used to overcome the impact of this noise and get a clear image, which in turn means that more x-rays are absorbed by the patient.

The total x-ray dose, and therefore the number of emitted photons, must be minimized if x-rays are to be used for screening for tumors. "Turnstile" photon emitters, which release individual photons on demand in a controlled manner, allow users to know when and how many photons are released.

A simple Monte Carlo simulation has been devised which shows that eliminating one of the random components involved in radiological imaging would produce more accurate images with significantly lower doses of radiation. This simulation assumes monochromatic (single frequency) x-rays.

2. MATERIAL AND METHODS

Simulation

Grayscale images are used in the simulation to serve as test objects through which simulated x-rays are sent. The input images consist of $N \times N$ arrays of pixels and are placed perpendicular to a source of parallel x-ray beams (Fig.1). Each pixel is stored as a byte and has a value between 0 and 255. This value is interpreted as the pixel's x-ray opacity, which is essentially a measure of how easily a photon can go through a region without being absorbed. The gray value 0 corresponds to transparency to x-rays, and 255 to near opaque. In our images, 0 is shown as black and 255 as white.

Beer's Law is used to determine the probability of a photon being absorbed. It is calculated by

$$P = e^{-R\mu} \quad (1)$$

where R is the thickness of the pixel and μ is its opacity. The simulation assumes that the thickness remains the same for the entire image and somewhat arbitrarily uses a value of

$$R = -\frac{\ln(0.5)}{128} \quad (2)$$

The value of R can be chosen arbitrarily. The above value assigns a probability of absorption of $P = 0.5$ when the pixel opacity is $\mu = 128$, which is the median pixel value. This value is used so as to give a wider range of probabilities for the single slab considered. A larger number R would increase the probability of a photon being absorbed by the object, and a smaller value for R would reduce this probability.

The simulation program therefore interprets the input image's array of values as a representation of a three dimensional object with thickness R , as shown in Fig. 1. This configuration is equivalent to that of specimen radiography.

A known number of photons go into each pixel, and a trial is performed for each photon assigned to that pixel. Each trial consists of generating a uniformly distributed random number between 0 and 1. The photon is counted as having gone through the pixel only if the random number is less than the probability of absorption, as calculated in Equation (1). The output is an $N \times N$ matrix of values, where each value represents the number of photons passing through the slab unabsorbed. These resultant values are scaled to between 0 and 255 by finding the average value of the image and scaling this to a reference value of 128. Any value on this new scale with a value of

over 255 is set to 255. In order to facilitate comparison between input and output, and since it is counterintuitive to represent transparency with black, the simulation then takes the negative of the matrix (255 minus the gray value) and interprets this final result as a grayscale image. The negative of the output images taken.

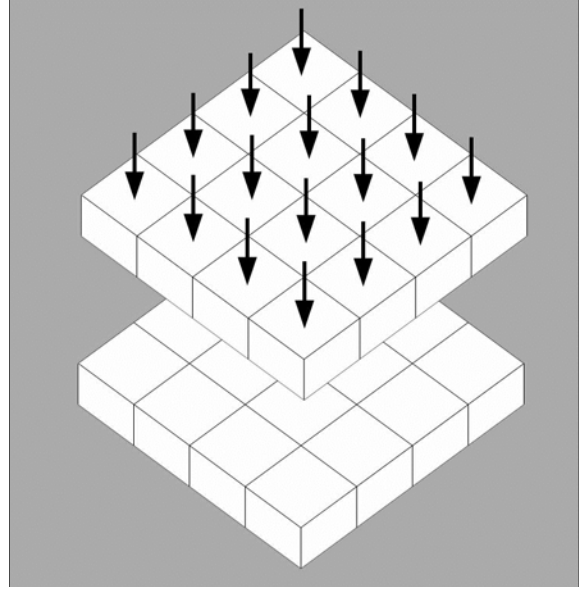


Fig. 1. X-ray simulation setup.

Simulations were performed involving various numbers of photons. The number of photons chosen range between 1 and 5000 in semi-logarithmic increments.

The difference between the turnstile and Poisson simulations is the number of photons assigned to each pixel, which corresponds to the number of photons per beam. For a turnstile emitter, the number is set by the programmer and remains the same for all pixels. For a Poisson emitter, the number is chosen from a Poisson distribution with the same number given as the mean.

Four different input images were used, however, only the breast input image, with $N = 256$, is shown in this paper (Fig.2).

The output images (Figures 2.a, 2b, 2c and 2d) are quantitatively compared using Mean-Square Error (MSE) and Entropy measures.

Turnstile emitters significantly outperform Poisson photon emitters for all of input images and for all image comparison techniques used at low doses, and then converge with the Poisson emitter images at higher doses.

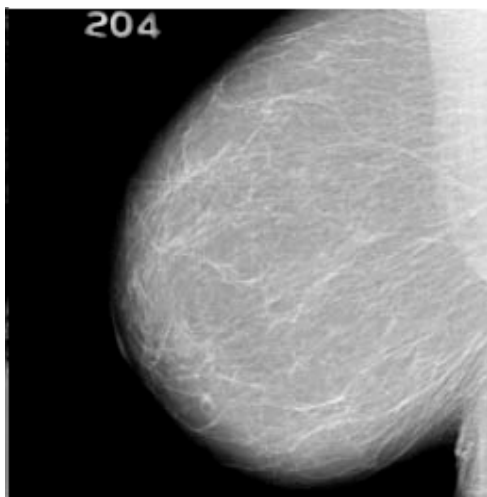


Fig. 2. Breast input image. Since the breast is compressed in this mammogram, it approximates the slab of Fig. 1.

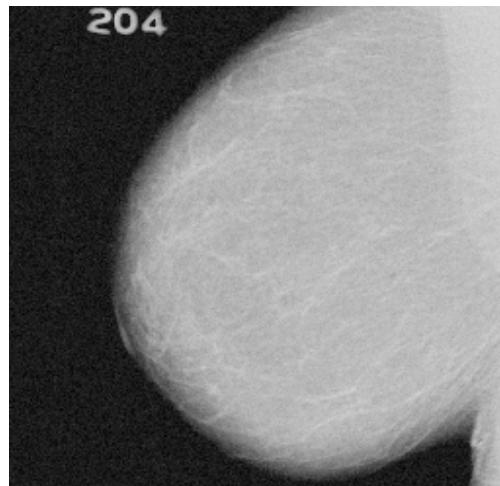


Fig. 2.c Breast output image. Poisson. 1000 photons/pixel

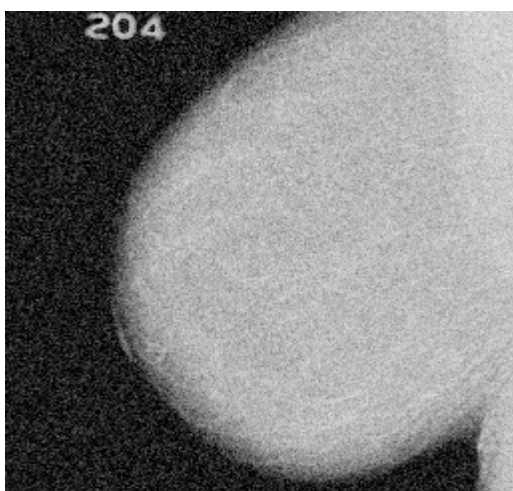


Fig. 2.a Breast output image. Poisson. 100 photons/pixel.



Fig. 2.d Breast output image. Turnstile. 1000 photons/pixel.

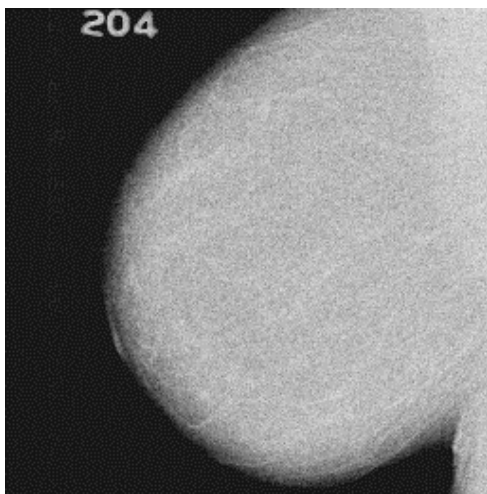


Fig. 2.b Breast output image. Turnstile. 100 photons/pixel.

3. RESULTS

3.1 Mean-Square Error

Mean-Square Error (MSE), also known as Euclidean distance or standard deviation, is the most common way of quantitatively comparing images. It involves finding the square of the difference between the corresponding pixel values of two scaled images, as expressed in the following formula:

$$MSE = \frac{1}{N^2} \sqrt{\sum_i^N \sum_j^N (Orig_{i,j} - Output_{i,j})^2} \quad (3)$$

The difference is squared in order to ensure that a negative difference between two corresponding pixels in the input and output images does not merely cancel out a positive difference between two other corresponding pixels. The Mean-Square Error involves dividing by the number of pixels (N^2) in order to prevent a larger image from having an inherently larger MSE.

After calculating the MSE of the turnstile and Poisson outputs, the ratio of Poisson to turnstile was calculated to show what dose would maximize the advantage of turnstile over Poisson. The graphs (Fig. 3.1a and Fig. 3.1b) show the MSE and MSE ratio, respectively, for the breast input.

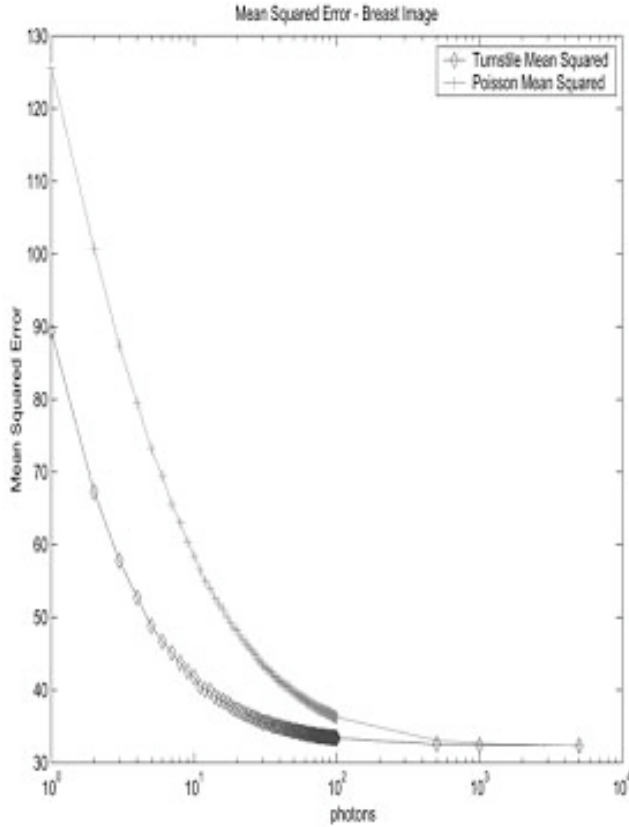


Fig. 3.1a. MSE for breast.

These graphs show a distinct advantage for turnstile emitters at low doses, with that advantage slowly decaying as larger number of photons are used and the random fluctuations of emitted Poisson photons becomes less significant. Unfortunately, while the Poisson and the turnstile emitters both converge to each other, they do not converge to the original image, as would be expected at high doses. This is due to the way both output images were normalized. This point will be elaborated upon later.

A significant result is the location of inflection point in Fig. 3.1b. The inflection point shows the dose at which the law of diminishing returns takes place. This suggests that

the advantage of turnstile emitters to Poisson emitters is due to the information collected at low doses and implies

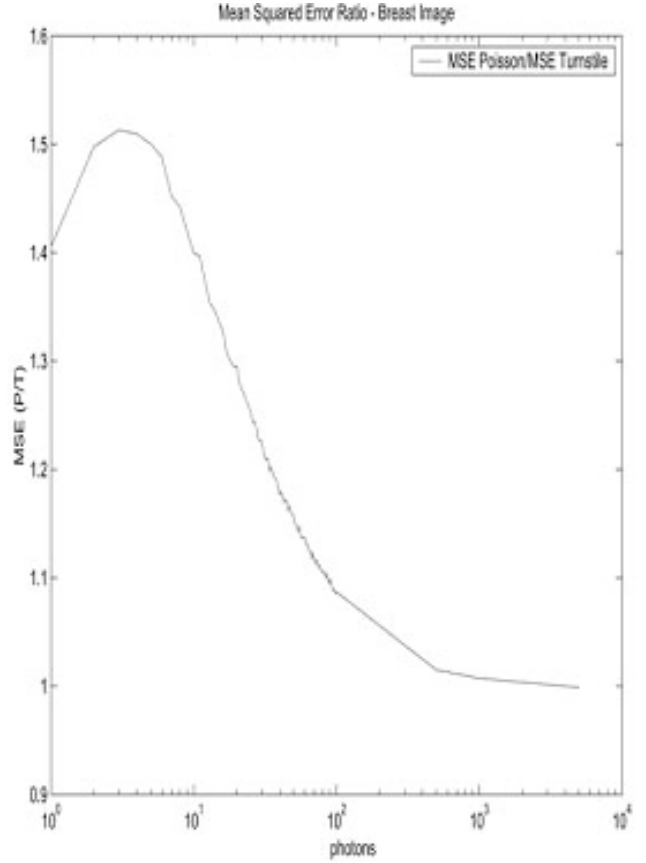


Fig. 3.1b. MSE ratio for breast, Poisson:turnstile photons

that turnstile emitters would be especially promising for use in CT imaging, in which relatively small numbers of photons must be used at each projection angle in order to minimize the total dose.

3.2 Entropy

Entropy basically measures the disorder of a system. The entropy of the output images was calculated with the following formula:

$$Entropy = \frac{-\frac{1}{N^2} \sum_i \sum_j [Output_{i,j} \ln(Output_{i,j})]}{\frac{1}{N^2} \sum_i \sum_j Output_{i,j}} \quad (4)$$

with the term in brackets evaluated as 0 when $Output_{i,j} = 0$. The graphs (Fig. 3.2a and Fig. 3.2b) show the entropy and entropy ratio, respectively, for the breast image.

These graphs agree with the MSE. Turnstile photons show a distinct advantage at lower numbers of photons, with the advantage decaying for higher numbers until the

plots converge and the apparent advantage of turnstile photons disappears. Interestingly, the entropy graphs show turnstile photons having the largest advantage at one photon per pixel.

Like the Mean-Square graph above, the entropy measure for both Poisson and turnstile images does not converge to one. This is also most likely due to the normalization method used.

4. THE IMPORTANCE OF NORMALIZATION

The number of output photons varies according to the number of emitted photons. Finding an appropriate method for normalizing the raw output data so that a grayscale image can be recreated for comparison to the original is the most difficult part of this simulation.

For this simulation, the original images are scaled so that each input image has a mean of 128. Although not essential, this scaling ensures that the input images have a wide range of gray values. The thickness of the image was also chosen to give an x-ray photon a 50% chance of going through areas in the object with opacity of 128 without being absorbed. This further ensured a broad range of data. The mean of the output matrix is scaled to 128; afterwards the negative (255 minus gray value) is taken. Values greater than 255 are truncated to 255. This “normalization” gives the output image with an artificially high number of pixels with the value 255.

In basing all output pixel values around the average output value, the simulation does not take into account that the probability of absorption as a function of pixel opacity has a natural logarithmically scaled distribution. As a result, the raw output data contains more low valued pixels than high ones, resulting in a mean pixel value that is noticeably lower than the median value. Therefore, the data, once scaled, would also be skewed to one end, with a very low probability of having pixel values at the other end of the spectrum.

5. FUTURE APPLICATIONS

5.1 Computed (Axial) Tomography

The most obvious extension of this simulation is to apply turnstile photons to computed tomography (CT) imaging. CT imaging is similar to x-ray imaging in that a x-ray machine emits photons through the object to be imaged. In CT the x-ray emitter revolves around the patient, acquiring “snapshots” called profiles, from multiple angles. The position and the outputs at these profiles are then fed into a computer algorithm, which uses this information to create a 3D image of the original object. Because x-rays are now emitted from a variety of angles and the total dose must remain as low as possible, fewer photons can be emitted for each profile than in standard x-ray imaging. Turnstile photon emitters, which

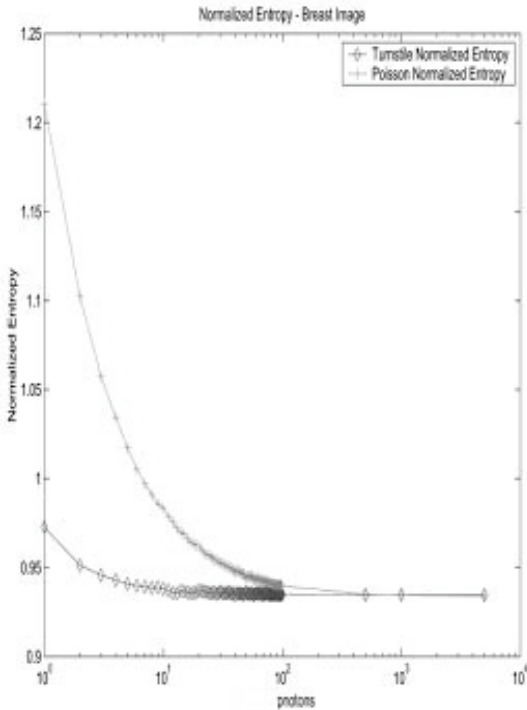


Fig. 3.2a. Entropy for breast

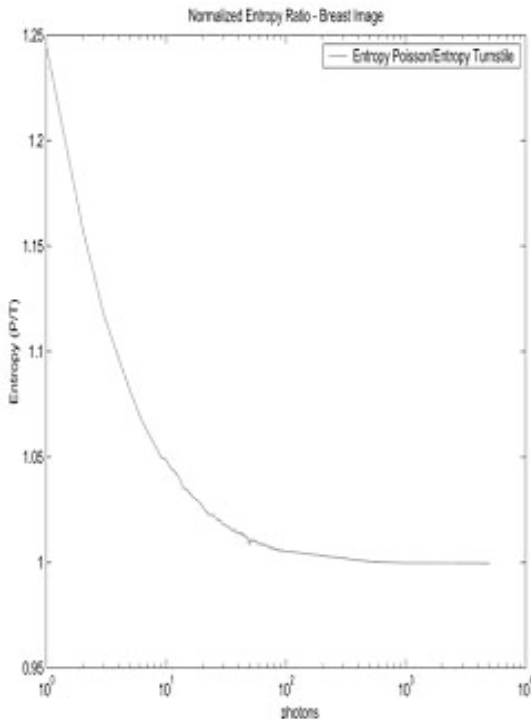


Fig. 3.2b. Entropy ratio for breast

provide significantly better images than Poisson photon emitters at low doses, are therefore much better suited to CT imaging than current emitters, and their superiority to Poisson emission for CT imaging may be significantly more pronounced than in the simple x-ray simulation shown here.

Turnstile photons, by virtue of their release being controlled, allow the user to count the number of photons emitted. From this amount, the number of photons detected could then be subtracted to give the total number of photons absorbed by the object, and this could be used to produce a CT algorithm that reconstructs images one photon at a time [5].

5.2 Multiphoton Microscopy and Low Light Level Imaging

Bright field microscopy with visible light involves focusing photons on a specimen. Some photons are absorbed or scattered, and the rest are used to form an image. In fluorescence microscopy, the illumination consists of ultraviolet (UV) photons, which excite fluorochrome molecules to emit visible light. When the specimen is a living cell, the UV can also damage it. To avoid this, two photons of exactly half the energy of an ultraviolet photon (i.e., in the infrared range, IR) can be used. For a Poisson emitter, the probability that two photons are in the same focus volume at the same time in order for the absorption to occur is low, unless the IR source is quite intense. It should be possible, however, to synchronize turnstile emitters to emit photons at the same time, allowing a much less intense source.

5.3 Nuclear Applications

This technique of emitting photons in synchrony with each other could be used in any number of applications to concentrate power at a single point. Taken to an extreme, it may even be possible to develop very high-powered lasers that would concentrate enough energy at a given point to set off a nuclear fusion reaction. Turnstile emitters may be able to initiate a reaction with much less energy than is currently required.

6. CONCLUSIONS

Regardless of the anomalies resulting from normalization choices, the methods chosen give quite favorable results for turnstile emitters when compared to Poisson emitters: turnstile emitters provide better resolution at lower doses.

These results imply that better images for a given dose could be achieved using turnstile photons.

Sivaramakrishna and Gordon [1] showed that finding and destroying tumors smaller than 4 mm in diameter could prevent cancer from spreading and thus reduce cancer deaths to a small fraction of current rates. Developing turnstile emitters for x-ray imaging could be pursued as a means to this end.

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