Communication

Interpolative algebraic reconstruction techniques without beam partitioning for computed tomography

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1 Introduction

ALGEBRAIC RECONSTRUCTION techniques (ART) were introduced (GORDON *et al.*, 1970) for solving what is known as the computed tomography problem. They are based on an intuitive approach of smearing back each projection of the estimate of the object's optical density with repeated corrections until an agreement with all the corresponding measured projections is reached.

All ART algorithms (ANDERSEN, 1989; GORDON *et al.*, 1971; GORDON, 1974; HERMAN *et al.*, 1973; OSKOUI-FARD *et al.*, 1988) have the same basis; the partition of the reconstruction matrix into a set of 'rays' or 'passages' through which the radiation traverses the object. Every radiation passage is represented by a projection value. The reconstruction is performed over a set of pixels generally chosen in a regular array. For every pixel to obtain an approximation for an unknown optical density function, we assign an estimate of its value based on the fraction of the pixel covered by the passage and the values of the projections for that particular passage. The process is repeated many times until a convergence criterion is met.

In order to make the calculations less cumbersome, in the parallel beam case, for every projection direction the passage width can be chosen so that one pixel centroid per row is encountered, except for the last row (GORDON, 1974); HERMAN *et al.*, 1973). The assumption that centroids can replace pixels simplifies the procedure, as for every projection direction we only need to find which centroids belong to which passage; i.e. if we disregard the edge or 'partial volume' effect (GORDON, 1974). This eases slightly the difficulty of calculating for every pixel and for every projection direction how much of the pixel is being covered, and by which of the passages. However, it means that during the reconstruction process, for every projection direction we have to calculate the width of the passage, as well as to record or recalculate which centroids belong to which passages.

We introduce a new method to relate the pixels to their projections which is more suitable for data from detectors of unchanging width, as occur in practice. The new method is moreover readily applicable to both parallel and diverging beam geometries.

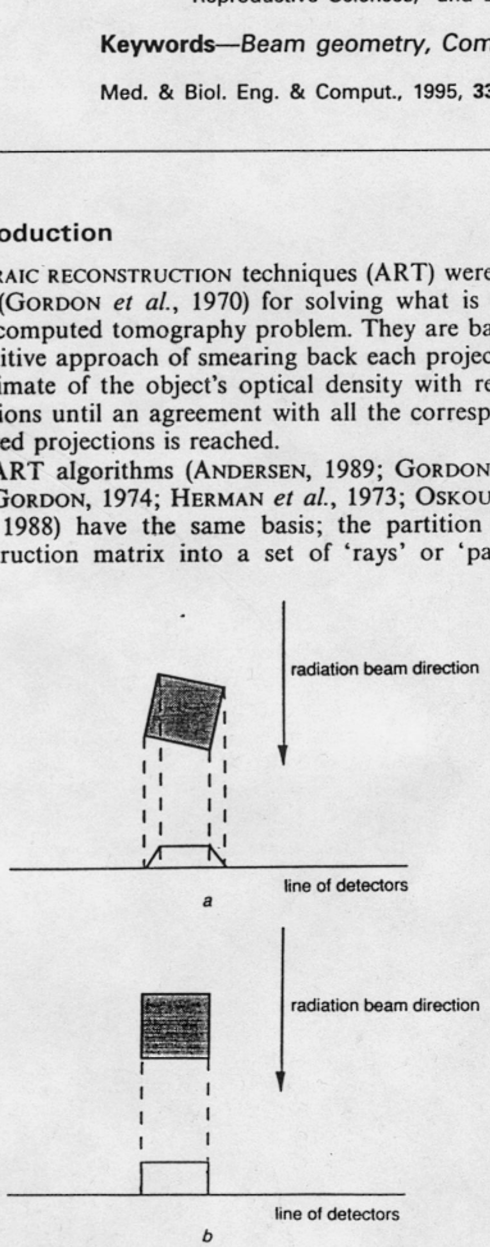

Fig. 1 *Estimation of pixel shadow on the line of detectors using parallel geometry beam with (a) fixed pixel and (b) rotated pixel interpolation*

2 Interpolative algebraic reconstruction technique (IART)

The new method that we introduce combines this novel technique to relate pixels to projections with the algebraic reconstruction techniques approach to reconstruct an object. IART is similar to the ART approach of throwing back each projection across the reconstruction matrix pixels from which it came to assign a new estimate of the unknown optical density function $f(\vec{r})$, where $\vec{r}$ represents a position vector in the object. The estimated value for $f(\vec{r})$ at each iteration is based on the discrepancy between the calculated and corresponding measured projections. The iterations are

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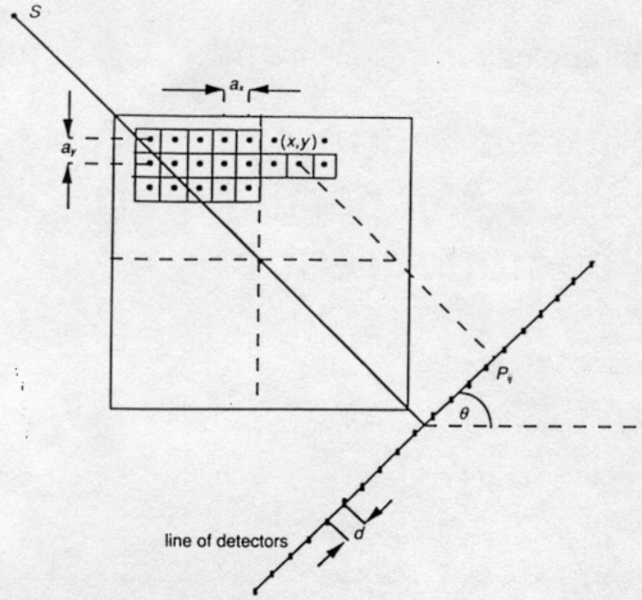

Fig. 2 *Line at angle θ represents line of detectors for projection direction $\theta + 90°$; S = X-ray source that scans over detectors forming parallel beams; a_x, a_y = distances between pixel centres in rows and columns of the back-projection matrix, respectively; d = detector width*

halted when the calculated and corresponding measured projections become approximately equal in value.

In a computed tomography scanner, when the radiation beam passes through an object or patient, it produces readings on a line of detectors behind the object. In order to find the relationship between the individual pixels of the reconstruction matrix and the line of detectors, we use the shadow cast by the pixel on the line of detectors (Fig. 1). By evaluating how much of the shadow is covered by which of the detectors, we estimate the contribution of the element to the detectors involved.

We can perform the calculations either by accurately estimating the shape of the shadow for every projection direction ('fixed pixel' interpolation), or by making an assumption that the shadow is of the same rectangular shape regardless of the projection direction (Fig. 1). The latter is equivalent to rotating the pixel so that it is aligned with the radiation beam direction; therefore we call it the 'rotated pixel' interpolation. With fixed pixel interpolation, for the parallel geometry beam the shadow changes its shape, from rectangular through trapezoid to triangular, depending on the angle of a projection's direction. With the rotated pixel interpolation, the shadow of a pixel is always the same rectangle. For a fan beam, with a fixed pixel interpolation, the shadow changes its shape from trapezoid to triangular, depending on the angle of a projection direction and the distance of a pixel from the central ray of the radiation beam. With the rotated pixel interpolation, the pixel shadow always has the trapezoid shape. For the sake of simplicity of calculation, we can approximate the latter by a rectangle. As the shapes of pixels are artifacts of our digitisation of continuous real-world images, consideration of alternative orientations or shapes for them seems reasonable.

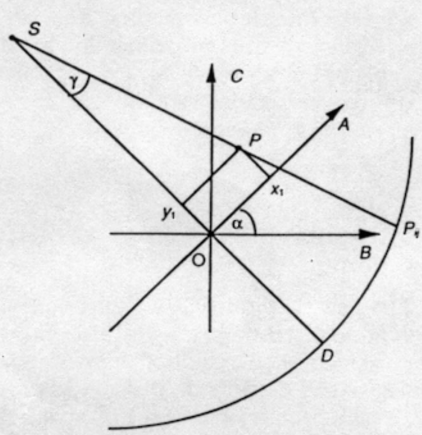

Fig. 3 *Geometry of fan beam scanning system; S = X-ray source; P = centre of (i, j)th pixel with co-ordinates (x, y) and (x_1, y_1) in COB and SOA co-ordinate systems, respectively; P_{ij} = centre of shadow cast by (i, j)th pixel on arc of detectors; SD = central ray of beam*

Below, for both parallel and diverging beams, we present the description of the rotated pixel interpolation as this is the one that we expect to be more popular. We show that results obtained using it are slightly worse than those obtained through the fixed pixel interpolation, whereas the computation time is shorter.

2.1 *Parallel beam*

We start by projecting the centre of the element onto the line of detectors (Fig. 2). Let us assume that the (i, j)th pixel in the back-projection matrix is described by a pair of co-ordinates (x, y), such as

$$x = ia_x$$

$$y = ja_y$$

where a_x and a_y represent the distance between pixel centres in the matrix, as in Fig. 2. Usually, $a_x = a_y = a$.

The position of the projection of the pixel centre onto the projection line at angle θ is equal to

$$P_{ij} = (y \sin\theta + x \cos\theta)/d \tag{1}$$

where d is the detector width.

The pixel contributes to all detectors covered by its shadow. In practice, it is common and justified to set the reconstruction element size a equal to the detector width d and further use it to describe the system dimensions. In this case, the pixel contributes to a maximum of two detectors that are the closest to point P_{ij}. The portion of the element density that adds to a reading of the detectors is calculated using the interpolation function

$$g_{ij}(u) = \begin{Bmatrix} 1 & \text{for} & |u - P_{ij}| = 0 \\ 1 - |u - P_{ij}| & \text{for} & 0 < |u - P_{ij}| \le 1 \\ 0 & \text{for} & |u - P_{ij}| > 1 \end{Bmatrix} \tag{2}$$

where u represents the centre of the detector and P_{ij} is the projection of the centroid of the (i, j)th pixel onto the detector line.

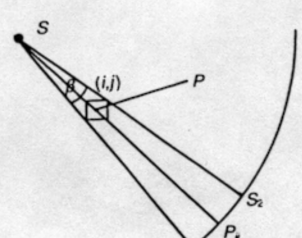

Fig. 4 *Shadow S_1S_2 cast by (i, j)th pixel onto detectors arc in fan beam scanning*