On Estimating the Relation Between Corporate Bond Yield Spreads and Treasury Yields

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Abstract

Using a simple theoretical framework, we issue two warnings with respect to the estimation of the yield spread - riskless rate relation. The first warning relates to Longstaff and Schwartz’s regression analysis applied to relative yield spreads. The second warning is closely related to Duffee’s (1998) caution against interpreting results of yield-spread studies based on data sets, such as that used by Longstaff and Schwartz (1995), consisting primarily of callable bonds. We provide strong support for both warnings using a data base of Canadian, investment grade, corporate bond indices containing a unique provision allowing to identify callable and noncallable indices and control for a possible coupon bias. Using these data, we also confirm Duffee’s (1998) assertion that the coupon bias accounts for the observed negative and weak relation between yield spreads and Treasury yields for his noncallable bond portfolios.
On Estimating the Relation Between Corporate Bond Yield Spreads and Treasury Yields

Longstaff and Schwartz (1995) present one of the most important and comprehensive models for the valuation of corporate bonds. The strength and richness of this model are inherent in its ability to incorporate features, important for the pricing of corporate bonds, which were previously modeled separately. Assuming that both the riskless rate and the value of the firm’s assets are stochastic, Longstaff and Schwartz’s (1995) analysis yields a two-factor model for defaultable bonds.

One of the most notable predictions of Longstaff and Schwartz’s (1995) model, as in other structural models with an asset-based default process, is that credit spreads are negatively related to the riskless rate. Ceteris paribus, an increase in the riskless rate implies a higher expected future value for the firm’s assets relative to the default threshold, and a lower risk-neutral probability in default. This ultimately results in a lower credit spread. Longstaff and Schwartz proceed to test the predictions of their two-factor model with respect to credit spreads.

The aim of this paper is twofold. First, through our theoretical and empirical analysis, we re-issue a warning originally stated by Duffee (1998) against interpreting results of yield-spread studies based on data sets consisting primarily of callable bonds. Duffee points out that the impact of callability likely confounds tests, such as those conducted by Longstaff and Schwartz (1995), seeking to isolate the impact of default risk. In the context of the credit spread - riskless rate relation, Duffee cautions that for callable bonds, following a given increase in the riskless rate, the yield of a corporate bond will increase similarly, and its price will fall. This will lower the probability of a call and result in an increase in the bond price and an offsetting drop in its risky yield. Consequently, the callable bond’s yield will rise less than the Treasury yield, resulting in a negative yield spread - riskless yield relation.

While some take note of Duffee’s warning (see for example, Helwege (1999), Jarrow and Turnbull (2000), and Papageorgiou and Skinner (2002)), many others overlook it (see for example, Duffie and Singleton (1999), Madan and Unal (2000), Das and Sundaram (2000), Collin-Dufresne and Solnik (2001), Lekkos and Milas (2001), and Das and Tufano
(1996) who refer to an earlier version of Duffee’s paper). Overlooking this warning, and implicitly assuming that the callability term is trivial may falsely lead to the conclusion that the credit risk drives the resultant negative yield spread - riskless rate relation found for callable bonds. This motivates us to reissue Duffee’s warning, and present a theoretical framework backed by empirical evidence that supports it.

The second warning we issue applies to Longstaff and Schwartz tests for the predictions of their two-factor model with respect to credit spreads. Throughout their empirical analysis, they use both absolute yield spreads and relative yield spreads, in the context of two different regression models. Relative credit spreads are used as a means for adjusting for differences in absolute spreads related to differences in the levels of interest rates. Our warning relates to Longstaff and Schwartz’s (1995) regression model applied to relative yield spreads. We demonstrate both theoretically and empirically that this model may generate spurious conclusions. In the empirical section of this paper, we use a data base of Canadian, investment grade, corporate bond indices containing a unique provision allowing for identification of callable and noncallable indices. Both warnings gain strong support from this data.

The paper is organized as follows. Section I describes Longstaff and Schwartz’s tests for the predictions of their two-factor model with respect to credit spreads. In Section II we provide the theoretical framework for our warnings. This theoretical framework describes the impact of both callability and default risks on the yield spread - riskless rate relation. We then turn in Section III to the empirical evidence supporting the warnings we issue. In Section IV we apply Duffee’s (1998) regression analysis to our data, and confirm his assertion about the impact of the coupon bias associated with corporate bonds. Section IV summarizes our results and discusses their implications for estimating the relation between corporate bond yield spreads and government yields.

I. Longstaff and Schwartz’s (1995) Regression Analysis

Longstaff and Schwartz test the predictions of their two-factor model with respect to credit spreads. In their tests, they apply a regression analysis to both absolute yield
spreads and relative yield spreads. For absolute yield spreads, they employ the following two-factor Ordinary Least Squares (OLS) regression model:

\[ \Delta S = a + b\Delta Y + cI + \varepsilon, \]  

(1)

where \( \Delta S \) is the monthly change in absolute yield spreads, \( \Delta Y \) is the monthly change in the 30-year Treasury yield, which proxies changes in interest rates, and \( I \) is the monthly return on the Standard and Poor’s industrial, utility, and railroad stock indexes (whichever applies), which proxies firm assets returns. For relative yield spreads, they employ the following two-factor OLS regression model:

\[ \Delta R = a + bPY + cI + \varepsilon, \]  

(2)

where \( \Delta R \) is the monthly change in relative yield spreads, and \( PY \) is the monthly percentage change in the 30-year Treasury yield.

Longstaff and Schwartz’s regression analysis yields results supporting the comparative statics of their model. In general, the regression coefficients, \( b \) and \( c \), are found significantly negative for both regression models, with stronger support provided by regression model (2). This implies that the predictions of their theoretical two-factor model of a negative credit spread - interest rate relation and a negative credit spread - asset return relation, are borne out in their data. In sections II and III below, we critique these tests in the context of the two warnings we issue.

II. The impact of Default and Callability Risks on Yield Spreads

A. Defining Absolute and Relative Yield Spreads

To analyze yield spreads of callable corporate bonds, one must account for the impact of both default and call risks. To this end, we extend the duration analysis framework of Fons (1990) and Jacoby and Roberts (2001), and derive an expression to describe the impact of both risks on the sensitivity of yield spreads to shifts in the riskless rate. The
yield to maturity of a risky and callable coupon bond \((y^c)\) is defined as the sum of the yield to maturity of a corresponding noncallable riskless coupon bond \((r)\), and a yield spread \((YS^c)\). The yield spread is a function of the risky bond’s time to maturity \((T)\), the risky coupon rate \((c)\), the coupon rate of a corresponding riskless bond \((c_f)\), the conditional probability of default \((p)\), the conditional probability of a call \((\phi)\), the call price set in the call provision \((E)\), and the call-protected period \((T_{CP})\). Thus the yield on the pure-discount, risky bond is given by:

\[
y^c = r + g(T, c, c_f, p, \phi, E, T_{CP}).
\]

The absolute yield spread for this zero-coupon risky callable bond \((YS^c = y^c - r)\) is given by:

\[
YS^c = g(T, c, c_f, p, \phi, E, T_{CP}),
\]  

(3)

and its relative yield spread \((R^c = \frac{y^c}{r})\) is given by:

\[
R^c = 1 + r^{-1}g(T, c, c_f, p, \phi, E, T_{CP}).
\]

(4)

In order to examine the relation between both spread measures and \(r\), we note that the conditional probabilities of default and call are related, among other parameters, to the riskless rate:

\[
p = f(r, ...), \quad \text{and} \quad 
\phi = h(r, ...).
\]

Since Longstaff and Schwartz (1995) work with both absolute and relative yield spreads, next we examine the sensitivity of each measure to shifts in the riskless rate.

\(B. \text{Absolute Yield Spread - Riskless Rate relation}\)
Given the above functional form, the sensitivity of absolute yield spreads to shifts in the riskless rate, is given by:

\[
\frac{\partial Y^c}{\partial r} = \frac{\partial g}{\partial p} \frac{\partial p}{\partial r} + \frac{\partial g}{\partial \phi} \frac{\partial \phi}{\partial r}. \tag{5}
\]

To render the analysis offered by equation (5) more manageable, we take a closer look at the terms on its right-hand side. We start with the first term, which refers to the risk of default. The term \(\frac{\partial g}{\partial p}\) is positive since following an increase in default risk, investors will demand a higher yield. The sign of \(\frac{\partial p}{\partial r}\) depends on the correlation between the level of riskless rates and default risk.

The second term on the right-hand side of equation (5) refers to the risk of a call. Since a higher probability of a call implies a higher call premium required by bondholders, and a higher risk-adjusted yield, the sign of \(\frac{\partial g}{\partial \phi}\) will always be positive. On the other hand, higher riskless rates imply a lower chance that the issuer will exercise the option provided by the call provision, and bondholders will demand a lower call premium. Hence, \(\frac{\partial \phi}{\partial r}\) will always be negative. Thus, the sensitivity of yield spreads to shifts in the riskless rate depends on how both default and call risks change with shifts in the riskless rate. In general, \(\frac{\partial Y^c}{\partial r}\) will be lower than \(\frac{\partial Y^c}{\partial r}\) of a corresponding non-callable bond, due to the negative sign of the callability term \(\frac{\partial g}{\partial \phi} \frac{\partial \phi}{\partial r}\).

The above analysis leads to the following equation describing absolute yield-spread sensitivity to shifts in the riskless term structure:

\[
\frac{\partial Y^c}{\partial r} = \frac{\partial g}{\partial p} \frac{\partial p}{\partial r} + \frac{\partial g}{\partial \phi} \frac{\partial \phi}{\partial r}. \tag{6}
\]

Equation (6) implies that the relation between yield spreads and the riskless rate can be negative whether the default-risk term takes negative, positive, or zero value. This may occur solely due to the negative sign of the callability term. This implication is consistent with Duffee’s (1998) empirical support for the importance of callability. For non-callable bonds Duffee finds a very weak relation between yield spreads and the Treasury yield. This
last finding implies that for non-callable bonds, $\frac{\partial Y_S}{\partial r}$ is approximately zero and therefore, the default term $(\frac{\partial g}{\partial p} \frac{\partial p}{\partial r})$ for investment-grade bonds must be approximately zero as well.

Note that Duffee (1998) still obtains a negative — although weak — relation between yield spreads and Treasury yields for his noncallable bond portfolios. Using an arithmetic exercise, Duffee shows that this result for noncallable bonds can be attributed to the coupon effect. Compared with corporate bonds, Treasury bonds will tend to be more sensitive to changes in Treasury yields due to their lower coupon rates. Therefore, a given change in the long-term Treasury yield will result in a smaller change in yields of corporate bonds, which may explain the observed weak negative relation.\(^5\)

The above analysis, together with Duffee’s (1998) findings for noncallable bonds, emphasizes that yield-spread sensitivity to shifts in the riskless rate is determined mainly by callability, rather than by the somewhat weak impact of the interaction between the bond’s default risk and the Treasury yield. It provides a theoretical backing to Duffee’s (1998) warning against interpreting results of yield-spread studies based on data sets consisting primarily of callable bonds, like the one used by Longstaff and Schwartz (1995).

Although some take note of Duffee’s warning (see for example, Helwege (1999), Jarrow and Turnbull (2000), and Papageorgiou and Skinner (2002)), others overlook it and interpret Duffee’s result to indicate a negative credit spread - riskless rate relation (see for example, Duffie and Singleton (1999), Madan and Unal (2000), Das and Sundaram (2000), Collin-Dufresne and Solnik (2001), Lekkos and Milas (2001), and Das and Tufano (1996) who refer to an earlier version of Duffee’s paper). Overlooking this warning, and implicitly assuming that the callability term is trivial may falsely lead to the conclusion that the default term, representing credit risk, is the one which drives the resultant negative yield spread - riskless rate relation found for callable bonds. This motivates us to reissue Duffee’s warning.

C. Relative Yield Spread - Riskless Rate relation

From equation (4), the sensitivity of relative yield spreads to shifts in the riskless rate, is given by
\[
\frac{\partial R^c}{\partial r} = r^{-1} \frac{\partial g}{\partial p} \frac{\partial p}{\partial \phi} + \frac{\partial g}{\partial \phi} \frac{\partial \phi}{\partial r} - r^{-2} g(\cdot). \tag{7}
\]

Equation (7) emphasizes the problematic nature of using relative spreads to study the yield spread - riskless rate relation (as in regression model (2)). For noncallable bonds (with \(\frac{\partial g}{\partial \phi} \frac{\partial \phi}{\partial r} = 0\)), with a zero default term (\(\frac{\partial g}{\partial p} \frac{\partial p}{\partial r} = 0\)) as implied by Duffee’s (1998) study, equation (6) shows that the absolute yield spread is insensitive to shifts in the riskless rate. However, based on equation (7), the relative spread for the same bond will be negatively related to the riskless rate: \(\frac{\partial R}{\partial r} = -r^{-2} g(\cdot) < 0\). Of course, this negative relation is borne out of the mathematical definition of the relative spread, rather than reflecting a meaningful economic relation. This explains the fact that Longstaff and Schwartz’s regression analysis based on regression model (2), produces stronger support for the comparative statics of their model, compared with the results of regression model (1).

The analysis based on equation (7) prompts us to issue a warning regarding Longstaff and Schwartz’s regression model for relative spreads (regression model (2) in our paper). If one overlooks Duffee’s (1998) warning, and studies callable bonds, one can expect to spuriously find a stronger negative yield spread - riskless return relation for relative spreads compared with absolute spreads.

However, when one does take Duffee’s warning into consideration, one can expect regression model (1) to yield no absolute yield spread - riskless rate relation, while regression model (2) is likely to yield a significant negative relation for relative spreads. Also, since the use of the relative spread introduces a negative structure into the model, one should expect regression \(R^2\)’s, produced by model (2) to be significantly higher than those yielded by model (1). The same can be said with respect to the absolute value of the \(t\) statistics for \(b\).

III. Evidence Supporting the Warnings

A. Data

Our sample is based on month-end yield-to-maturity data from the Scotia Capital Markets (SCM) investment-grade Canadian corporate bond indices, reported by Statistics
The SCM corporate bond indices are stratified into four different investment-grade rating categories: AAA, AA, A, and BBB. In this study, we use SCM’s long-term corporate bond indices (as motivated below), reported for the 08:1976-07:2001 25-year period. Data for the AAA index are available only until March 1993, as no bonds fit this category for the later period. A long-term $i$-rated corporate bond index, $i = \text{AAA}, \text{AA}, \text{A}, \text{BBB}$, consists of all bonds in SCM’s $i$-rated corporate bond universe, with remaining terms to maturity greater than 10 years.

Our yield spreads for these long-term indices are calculated with respect to the constant maturity, long-term Government of Canada index, reported by CANSIM. Following Longstaff and Schwartz (1995), to proxy firm assets’ returns we use the (continuously compounded) monthly return on the Toronto Stock Exchange 300 index.

Table I reports a summary of statistics for the time series of absolute and relative yield spreads stratified by credit rating. Similar to the statistics reported by Longstaff and Schwartz (1995) for their U.S. data, the means of both absolute and relative yield spreads monotonically increase as credit quality decreases for all indices. The same is true for the standard deviation of both yield-spread measures.

***Insert Table I here***

B. Data Characteristics

We feel that for studying the yield-spread sensitivity to the riskless rate, the SCM data are more appropriate than other data in three different areas related to callability, the coupon level effect, and effects arising from taxation. These advantages are discussed below.

B.1 Callability

To control for callability, Duffee (1998) suggests stratifying the data by forming two portfolios for each rating category, one consisting of only callable bonds, and the other of noncallable bonds. However, there may be a selection bias in callability related to risk differences between callable and noncallable bonds. For example, Bodie and Taggart (1978)
claim that firms with more growth opportunities are more likely to issue callable bonds. If the firm’s expectations of growth are confirmed, its shareholders can avoid sharing this good fortune with bondholders by simply having the firm’s managers call the entire bond issue. Berk, Green, and Naik (1999) show that the systematic risk of asset returns is related to the firm’s growth opportunities. This implies that stratifying bond data based on callability may create two risk classes within each rating category.

Canadian corporate bonds have a special feature which allows to mitigate the potential selection bias associated with callability. Most Canadian corporate bonds issued starting 1987, carry a unique call provision called the “doomsday” call provision. This call provision makes it possible to control for callability for some bonds, facilitating the study of a set of corporate bonds broader than just noncallable bonds. A doomsday call provision sets the call price at the maximum of the par value of the bond, or the value of the bond calculated based on the yield on a Government of Canada bond (with a matching maturity) plus a spread (the doomsday spread). Below, we demonstrate that BBB-rated Canadian bonds are always traded with yield spreads much wider than the doomsday spread set in their call provision. Thus, the exercise of the doomsday call for BBB-rated bonds will rarely cause financial damage to the bondholders, and these bonds may be considered economically noncallable.8

To substantiate this feature of Canadian corporate bonds, we collect doomsday spreads for all bonds carrying the doomsday call for each month during the 01:1993-12:1999 period, from the Financial Post Corporate Bond Record. Since in this study we focus on SCM’s long-term bond indices, we limit our sample to data corresponding to bonds with maturity greater than ten years. We stratify our data by credit rating, and for every month, we calculate the average and standard deviation of doomsday spreads across bonds within each rating category. Since data for the AAA bonds are unavailable for most of the sample period, we obtain statistics only for AA, A, and BBB rated bonds.

To determine the moneyness of the doomsday call provision, one must compare the doomsday spread to the yield spread. In Figure 1, for each rating category, we plot the yield spread on the corresponding long-term index (S), the average doomsday spread (µ), the average doomsday spread plus one standard deviation (µ + σ), and the average
doomsday spread plus two standard deviations \((\mu + 2\sigma)\). In Panel A of Figure 1, we see that for AA-rated bonds the yield spread is lower than the average doomsday spread in a large number of cases, and it is almost always lower than the doomsday spread plus one standard deviation. Similarly, in Panel B of Figure 1 we see that the yield spread of A-rated bonds is often lower than the average doomsday spread, and in a significant number of cases it is lower than the doomsday spread plus one standard deviation.

Based on this 7-year sample it is clear that the probability of the doomsday call being in the money for AA- and A-rated bonds is substantial, with that of AA-rated bonds being significantly larger. In Panel C of Figure 1 we see that the yield spread of BBB-rated bonds is always significantly higher than the average doomsday spread plus two standard deviations. Thus, based on our sample, we conclude that the probability of the doomsday call being in the money for a BBB-rated bond is virtually zero, and thus BBB-rated bonds are economically noncallable.

The presence of the doomsday call has important implications for studying the yield spread - riskless rate relation. For a standard call provision carried by most U.S. corporate bonds, lower riskless rates imply a higher probability of the issuer calling the bond. For the doomsday call of BBB-rated bonds, the effect of lower riskless rates is defused by the call price floating upwards. In the context of equation (6), this implies that \(\frac{\partial \phi}{\partial r} = 0\). For BBB-rated bonds, the callability term in equation (6) disappears, leaving default as the sole factor that may affect the yield spread - riskless rate relation. Thus, the existence of the doomsday call provides a useful instrument for the isolation of the effect of default risk and its significance.

This result allows us to control for callability for the long-term BBB-rated SCM bond index, during the later period of our 25-year sample. To determine the duration of this period, for each month during the 01:1993-12:1999 7-year period we count the number of corporate bonds issued with a doomsday call provision, the number of bonds issued with a standard call provision, and the number of noncallable bonds, as reported in the Financial Post Corporate Bond Record. In Figure 2 we plot the proportion of each of the above
categories calculated with respect to the total number of bonds within each month. As most Canadian corporate bonds issued after 1986 carry a doomsday call, in Figure 2 we see that the proportion of such bonds increases considerably during the 7-year period, from 22.82 percent in 01:1993 to 47.48 percent in 12:1999. At the same time, the proportion of bonds carrying a standard call provision decreases dramatically, from 41.03 percent in 01:1993 to 8.25 percent in 12:1999. The proportion of noncallable bonds fluctuates between 34.46 percent and 47.29 percent.

***Insert Figure 2 here***

The above proportions are calculated for the entire Canadian corporate bond universe as covered by the Financial Post Corporate Bond Record. One can expect the proportion of long-term doomsday bonds, with over ten years to maturity, to be much higher compared to that of medium-term and short-term bonds. Since most Canadian corporate bonds are issued with 10 to 20 years to maturity, it is more likely that the number of newly issued short-term and mid-term bonds in SCM's mid-term and short-term indices is dominated by the number of seasoned bonds. Thus, for those indices, it is more likely that most bonds are those originally issued prior to 1987 with the standard call provision.

CANSIM reports that as of 01:1987, the weighted average maturity for SCM's long-term AA, A, and BBB indices, is 14.61 years, 14.80 years, and 13.34 years, respectively. Note that these maturities represent the maturity of the last Canadian corporate bonds issued with a standard call provision prior to 1987. Thus, four years or so later, the “average” bond, issued originally with a standard call provision, is expected to become a medium-term bond, with maturity below ten years. Since these are averages, to be safe, we feel that it is reasonable to wait eight years instead, and to assert that starting 01:1995, the vast majority of bonds included in SCM’s long-term bond indices carry a doomsday call rather than a standard call provision.

Following the above discussion, we conclude that SCM’s long-term bond indices are the indices suitable for our study, and thus we discard the mid-term and short-term indices. Furthermore, we conclude that the 01:1995-07:2001 period, in which these long-term indices
consist mainly of bonds carrying a doomsday call, is an adequate estimation period to control for the callability of the BBB-rated index.

\textit{B.2 The Coupon Level Effect}

Recall that Duffee (1998) finds a negative — although weak — relation between yield spreads and Treasury yields for his noncallable bond portfolio. Duffee attributes this negative relation to the coupon level effect. Historically, SCM imposed constraints on the range of coupon rates permitted for corporate bonds to be included in its indices. These constraints were designed to eliminate the coupon level effect from the yield spread of the included bonds over the yield on Government of Canada bonds, so that this spread is as close as possible to the true yield spread. At present, corporate bonds have relatively lower coupon rates, and these constraints are no longer necessary. SCM has decided to drop these restrictions following the objective of better representing the Canadian secondary bond market. Therefore, the SCM time-series data are also suitable to control for the coupon level effect, since corporate bonds carry coupons with levels comparable to those of Government of Canada bonds.

\textit{B.3 Effects Arising from Taxation}

By using our Canadian bond data, we also control for tax effects arising from the different tax rates, which apply for U.S. corporate and Treasury bonds. Duffee notes that U.S. corporate bonds are subject to taxation at the federal, state, and local levels, while U.S. Treasury bonds are subject only to federal tax. He demonstrates that this tax differential is a factor affecting the yield spread - riskless rate relation.\textsuperscript{10}

Elton, Gruber, Agrawal, and Mann (2001) decompose yield spreads (calculated based on their estimated zero curves) for U.S. investment grade corporate bonds into three components: default-risk premium, state-tax premium, and systematic-risk premium. Their analysis shows that the state-tax premium is significantly more important relative to the default-risk premium. Thus, they warn against ignoring the tax differential when studying corporate bond yield spreads. Canadian corporate and Government of Canada bonds are subject to an identical tax rate. This means that taxation does not play a role in the estimated relation for Canadian corporate bonds.
Another advantage of using the Canadian bond data relates to both the level of coupons and the tax system. Constantinides and Ingersoll (1984) show that a dynamic bond trading strategy, aimed at minimizing tax liabilities, produces bond prices significantly higher than when using a buy-and-hold strategy. They attribute this difference in value to the tax-timing option. Jordan and Jordan (1991) provide strong evidence supporting the existence of a tax-timing option for U.S. Treasury bonds.

Prisman, Roberts, and Tian (1996) demonstrate that, given the different tax treatment for bonds in Canada, the tax-timing option in Canada is unlikely to have an economic value to bond traders. In addition, they show that when the range of coupon rates in the portfolio contracts, the value of the tax-timing option will be even lower. Given the constraints imposed on bonds to be included in our SCM indices in the past, and the tight range of coupon rates at the present, the value for the tax-timing option for these indices is expected to be even lower, and its impact on the estimated relation is minimal.

In summary, analyzing yield spreads using the SCM Canadian corporate bond indices, has the advantage of controlling for three factors: callability, the coupon level effect, and effects arising from taxation. Such an analysis provides a clearer picture of the role of the default-risk adjustment in measuring the sensitivity of investment-grade bond yield spreads to changes in the riskless rate.

C. Regression Methodology

In most cases, we find that our SCM data set is characterized by the autoregressive nature of the OLS residuals of regression models (1) and (2). Moreover, the order of the autoregressive process for the residuals varies across the indices. To determine the correct autoregressive order, we apply a stepwise autoregression method, which initially fits our regression model with 5 autoregressive lags, and then sequentially removes autoregressive parameters. This process continues until all remaining autoregressive parameters have significant t-tests at the 5 percent level of confidence.

In a significant number of cases, where the OLS residuals follow an autoregressive process, they also exhibit a nonconstant volatility consistent with a GARCH(1,1) process. Applying the Lagrange multiplier (LM) test and the Portmanteau Q-test, the results of both LM and Q statistics indicate significant first-order heteroscedasticity. In those
instances, we apply a maximum-likelihood estimation procedure for a combined autoregressive model, with orders as determined by the stepwise autoregression method, and a GARCH(1,1) model. When the OLS residuals only follow an autoregressive process, we use the Yule-Walker method (sometimes called the two-step full transform procedure). This estimation method is adapted to the autoregressive orders we obtain from the stepwise autoregression. Finally, when the OLS residuals are homoscedastic and do not follow an autoregressive process, we use OLS to estimate the two regression models.

Longstaff and Schwartz (1995) estimate the coefficients of regression models (1) and (2) using OLS. Although, as argued above, this estimation methodology is unsuitable for our SCM data set, for the sake of comparison of our results with those obtained by Longstaff and Schwartz, we also report the OLS estimates for our data. Applying a single estimation method (namely OLS) to all indices also enables us to draw a comparison across the SCM indices, without risking confusing differences borne out in the data, with differences arising from different estimation methods. Note that one’s conclusions when analyzing the results are insensitive to the estimation method. Thus, our focus in the following discussion is on the results of the OLS estimation. We report the results of our OLS estimation for regression models (1) and (2) in Tables II and III below. The results of the estimation method accounting for autoregressive and/or heteroscedastic residuals are reported in Tables A.1 and A.2 in the appendix.

\section*{D. Regression Results}

\subsection*{D.1 Results Supporting Duffee’s (1998) Warning}

In Table II, we report the OLS estimates for regression model (1). Panel A outlines the estimates for our entire sample, covering the 08:1976-07:2001 25-year period. Corporate bonds carrying a standard call provision dominate the data during this sample period. Thus, one can expect the impact of callability on the results to be significant for that period. In particular, equation (6) predicts that due to callability, the estimate for \( b \) in regression model (1) will tend to be negative.\footnote{\textit{***Insert Table II here***}}
The results of our estimation of regression model (1), reported in Panel A of Table II, are in agreement with the results of Longstaff and Schwartz (1995) for their U.S. bonds. Our estimated coefficients $b$ are negative and statistically significant for all indices during the 08:1976-07:2001 estimation period. As in Longstaff and Schwartz (1995), the magnitude of the estimates of $b$ is economically significant for the evolution of yield spreads, and the coefficient $b$ monotonically decreases with credit quality. Also in agreement with Longstaff and Schwartz (1995), Panel A of Table II reports that the estimates of $c$ are all negative and statistically significant.\textsuperscript{14} The coefficient $c$ is also found to decrease monotonically with credit quality, and is economically significant. This supports the importance of Longstaff and Schwartz’s (1995) asset factor, i.e., \textit{ceteris paribus}, higher firm values result in lower probability of default and consequently lower yield spreads.

Based on the above results for $b$, if one ignores the impact of callability, one may conclude that the negative and significant sign of the default term in equation (6) is the reason for the negativity of $b$. However, following Duffee’s (1998) results we believe that this result is due to the negative impact of the callability term. To test this hypothesis, we apply regression model (1) to all indices for the 01:1995-07:2001 period, in which our long indices are expected to be dominated by bonds carrying the doomsday call.

This allows us to control for callability for the BBB index. Recall that for BBB-rated bonds the doomsday call will always be out of the money, making them economically non-callable. Based on equation (6), this leaves the default term as the only factor potentially affecting the sign of $b$ for the BBB index. Based on their findings, which are similar to our results for the entire 25-year sample period, Longstaff and Schwartz (1995) conclude that the interest-rate factor is more important for lower-rated bonds. Given that the BBB index is the lowest-rated index in our sample, one can expect the default term, representing the interest-rate factor in Longstaff and Schwartz (1995), to have the strongest impact on the yield spread - riskless rate relation estimated for this index.

Panel B of Table II reports our results for the 01:1995-07:2001 period. Our estimated coefficient $b$ for the BBB index is statistically insignificant.\textsuperscript{15} This result implies no relation between credit spreads and the riskless rate for the economically non-callable corporate bonds rated BBB. In the context of equation (6), this result indicates that the default
term, which represents Longstaff and Schwartz’s interest-rate factor, is trivial for BBB-rated bonds, for which it is expected to be most important within our sample. Contrasting these results with those reported in Panel A of Table II for the same index, it becomes clear that what drives the negative sign of $b$ is the negative impact of the callability term.

In Figure 1 we show that the probability of a call for the doomsday call provision in the AA- and A-rated indices is significant. Thus, one should expect the doomsday call for these indices to induce a negative yield spread - riskless rate relation, as in the case of bonds carrying the standard call provision. This negative relation is confirmed by the results reported in Panel B of Table II. Our estimated coefficients $b$ are negative and statistically significant for the economically callable AA- and A-rated indices during the 01:1995-07:2001 estimation period.

These findings provide a strong and clear-cut endorsement for Duffee’s (1998) warning against interpreting results of yield-spread studies based on data sets consisting primarily of callable bonds. If one overlooks the dominant impact of callability for investment-grade bonds, which generates a negative yield spread - riskless rate relation, one may spuriously conclude that this result is due to the interest-rate factor represented by the default term in equation (6). Thus, one should always control for callability, either by studying only noncallable bonds when using U.S. corporate bond data, or by taking advantage of the doomsday call attached to most Canadian corporate bonds, and is economically noncallable for certain classes of credit ratings.

Figure 1 demonstrates that the probability of a call is significantly higher for AA-rated bonds compared with A-rated bonds. Thus, one may expect the negative impact of the callability term on the sign of $b$ to be greater for the AA index than for the A index. The results reported in Panel B of Table II support this hypothesis. Above, we saw that for the entire 25-year sample period the estimated coefficient $b$ monotonically decreases with credit quality. For the 01:1995-07:2001 period this monotonicity is reversed in line with the moneyness of the doomsday call.

Finally, the estimates for the coefficient $c$ for the 01:1995-07:2001 period in Panel B of Table II are still all negative and statistically significant. In general, the coefficient $c$ decreases with credit quality, and is similar in magnitude to that estimated for each index.
for the entire 25-year sample period. This clearly shows that Longstaff and Schwartz’s (1995) asset factor is robust.

**D.2 Results Supporting the Warning Regarding Regression Model (2)**

Table III outlines the OLS estimates for regression model (2). Panel A reports the estimates for our 25-year sample, covering the 08:1976-07:2001 period. Recall that data during this period are dominated by callable bonds, carrying a standard call provision. Since regression model (1) indicates a negative yield spread - riskless rate relation, then by equation (7) one may expect regression model (2) to provide even stronger support for such negative relation, i.e., higher absolute $t$ statistics for $b$ and regression $R^2$’s in regression model (2).

***Insert Table III here***

Compared with the results reported for in Table II, the results reported in Panels A and Panel B of Table III for regression model (2), indicate that the $t$ statistics for the coefficient $b$ is always higher when one uses the relative spread. Also, the regression $R^2$’s experience a significant increase when regression model (2) is applied, which suggests that this regression model introduces a negative structure into the data. Focussing on the economically noncallable BBB index during the 01:1995-07:2001 period, it is interesting to note that although we find no yield spread - riskless rate relation for absolute spreads, when relative spreads are used instead, $b$ becomes statistically negative.17

Thus, the results clearly support our warning regarding the use of regression model (2). As expected, when one overlooks Duffee’s (1998) warning, and studies callable bonds, one spuriously finds a stronger negative yield spread - riskless return relation for relative spreads compared with absolute spreads. When one takes Duffee’s warning into consideration, and studies only noncallable bonds, like our BBB index during the 01:1995-07:2001 period, one obtains no absolute yield spread - riskless rate relation, but a significant negative relation for relative spreads. Thus, when studying relative spreads, one may spuriously conclude
that an observed negative relative spread - riskless rate relation is due to the interest-rate
factor.

Finally, the estimates for the coefficient $c$ under regression model (2) for the entire
25-year sample period (Panel A of Table III) and for the 01:1995-07:2001 period (Panel B
of Table III) all remain negative and statistically significant. As in the analysis of absolute
spreads, the last result for relative spreads demonstrates that Longstaff and Schwartz’s
(1995) asset factor is robust.

IV. Duffee’s (1998) Regression Approach

Duffee (1998) uses a regression approach different from that of Longstaff and Schwartz
(1995). He regresses spread changes on changes both in the short yield and in a term
structure slope variable. Duffee finds evidence that changes in yield spreads on callable
bonds are strongly negatively related to changes in Treasury yields. On the other hand,
for noncallable bonds, he finds a weak, although still negative, relation. Duffee attributes
the result for noncallable bonds to the coupon effect.

Kamara (1997) presents evidence that the slope of the riskless term structure is pos-
itively related to expected economic growth. This finding implies a negative relation be-
tween default risk and changes in riskless rates. This is a possible alternative explanation to
the negative relation found for noncallable bonds. Our SCM time-series data are suitable to
control for the coupon effect, since corporate bonds carry coupons with levels comparable
to those of Government of Canada bonds. Thus, applying Duffee’s regression analysis to
our data clarifies the reason for the observed negative relation for U.S. noncallable bonds.

Following Duffee (1998), we estimate the following regression model for every index:

$$
\Delta S = \beta_0 + \beta_1 \Delta Y_{T-bill} + \beta_2 \Delta Slope + \varepsilon,
$$

where $\Delta S$ is the monthly change in absolute yield spreads, $\Delta Y_{T-bill}$ is the monthly change
in the yield on a three-month Government of Canada Treasury bill, and $\Delta Slope$ is the
monthly change in the spread between the constant maturity long-term Government of
Canada index and the three-month Treasury bill yield.
Table IV outlines the estimates for regression model (8). As in Section III, in some cases, we find that our SCM data set is characterized by the autoregressive nature of the OLS residuals of regression model (8). The order of the autoregressive process for the residuals varies across the indices. To determine the correct autoregressive order, we apply a stepwise autoregression method. In a number of cases, where the OLS residuals follow an autoregressive process, they also exhibit a nonconstant volatility consistent with a GARCH(1,1) process. In those instances, we apply a maximum-likelihood estimation procedure for a combined autoregressive model and a GARCH(1,1) model. When the OLS residuals only follow an autoregressive process, we apply the Yule-Walker method. Finally, when the OLS residuals are homoscedastic and do not follow an autoregressive process, we use OLS to estimate regression model (8). 18

Panel A of Table IV outlines the estimates for our entire sample, covering the 08:1976-07:2001 25-year period. Recall that corporate bonds carrying a standard call provision dominate the data during this sample period. As expected, the relation between yield spreads and both the three-month bill yield and the slope of the riskless term structure is significantly negative for all ratings. In general, both slope coefficients monotonically decrease with credit quality. These results agree with Duffee’s (1998) results for his callable U.S. bond portfolios.

Panel B of Table IV reports our results for the 01:1995-07:2001 period. For the economically callable AA- and A-rated indices, both slope coefficients are negative and statistically significant. Recall that BBB-rated bonds are economically noncallable during this period. Our estimated slope coefficients for the BBB index are both statistically insignificant. Applying regression model (8) to his U.S. bond data, Duffee still finds both slope coefficients to be significantly negative, although weak. He attributes this result to the coupon level effect. Our results support this assertion. Since this coupon bias is not prevalent in our SCM data, the negative sign for both slope coefficients disappears.

Recall that Figure 1 demonstrates that the probability of a call is significantly higher for AA-rated bonds compared with A-rated bonds. The results reported in Panel B of
Table IV are in line with this hypothesis. While for the entire 25-year sample period the estimated slope coefficients monotonically decrease with credit quality. For the 01:1995-07:2001 period, this monotonicity is reversed in line with the moneyness of the doomsday call. Thus, applying Duffee’s (1998) regression analysis to our data strengthens the hypothesis that callability drives the negative sign of both slope coefficients.

V. Summary and Conclusions

Longstaff and Schwartz (1995) present a significant model for the valuation of corporate bonds, which accounts for two stochastic factors. These factors are the short interest-rate factor and the firm’s asset value factor. One of the most notable predictions of this model, as in other structural models with an asset-based default process, is that credit spreads are negatively related to the riskless rate. Longstaff and Schwartz test this prediction along with their predicted negative impact of the asset factor on credit spreads.

In this paper we issue two warnings regarding the estimation of the yield spread - riskless rate relation. The first warning relates to Longstaff and Schwartz’s regression analysis applied to relative yield spreads. We demonstrate both theoretically and empirically that this model may lead to spurious conclusions. The second warning is closely related to Duffee’s (1998) work. Duffee warns against interpreting results of yield-spread studies based on data sets, such as that used by Longstaff and Schwartz, consisting primarily of callable bonds.

Although some take note of Duffee’s warning (see for example, Helwege (1999), Jarrow and Turnbull (2000), and Papageorgiou and Skinner (2002)), many others overlook it (see for example, Duffie and Singleton (1999), Madan and Unal (2000), Das and Sundaram (2000), Collin-Dufresne and Solnik (2001), Lekkos and Milas (2001), and Das and Tufano (1996) who refer to an earlier version of Duffee’s paper).

Our theoretical framework demonstrates that an observed negative yield spread - riskless rate relation originated from the impact of callability, can be spuriously attributed to the impact of the interest-rate factor. We provide strong support for both warnings using a data base of Canadian, investment grade, corporate bond indices containing a unique provision allowing to identify callable and noncallable indices and eliminate coupon effects.
Using these data, we confirm Duffee’s (1998) assertion that the coupon bias is the reason for the observed negative – although weak – relation between yield spreads and Treasury yields for his noncallable bond portfolios.
Appendix

***Insert Table A.1 here***

***Insert Table A.2 here***
References


Table I
Summary Statistics for Yield Spreads in SCM Long-Term Corporate Bond Indices for the August 1976 to July 2001 Period

The yield spread is the difference between the yield on a long-term index and the yield on the constant maturity, long-term Government of Canada index. The relative spread is the ratio of the yield on a long-term index to the yield on the constant maturity, long-term Government of Canada index. Data for the AAA indices are available only until March 1993.

<table>
<thead>
<tr>
<th></th>
<th>No. of Observations</th>
<th>Mean of Credit Spread</th>
<th>Std. Dev. Of Credit Spread</th>
<th>Mean of Relative Spread</th>
<th>Std. Dev. Of Relative Spread</th>
</tr>
</thead>
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<tr>
<td>AAA</td>
<td>200</td>
<td>0.5761</td>
<td>0.3050</td>
<td>1.0559</td>
<td>0.0331</td>
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<tr>
<td>AA</td>
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<td>0.3184</td>
<td>1.0716</td>
<td>0.0434</td>
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<td>A</td>
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<td>0.8612</td>
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<td>1.0980</td>
<td>0.0548</td>
</tr>
<tr>
<td>BBB</td>
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<td>1.5359</td>
<td>0.7916</td>
<td>1.1842</td>
<td>0.1266</td>
</tr>
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</table>
Table II reports the results of the OLS estimation of regression model (1) in which the dependent variable is the monthly change in the absolute yield spread. This regression model is of the following form:

$$\Delta S = a + b\Delta Y + cI + \varepsilon,$$

where $\Delta S$ is the monthly change in the absolute yield spread, $\Delta Y$ is the monthly change in the yield of the constant maturity, long-term Government of Canada index, and $I$ is the monthly return on the Toronto Stock Exchange 300 index. $t$-values are in parentheses. Panel A reports the estimates for the entire sample, covering the 09:1976-07:2001 25-year period. Data during this sample period are dominated by corporate bonds carrying a standard call provision. Panel B outlines the results for the 01:1995-07:2001 sub-period, in which bonds carrying the doomsday call are expected to dominate all indices.

Table II: Regressions of Changes in Absolute Yield Spreads of SCM Long-Term Corporate Bond Indices on Changes in the Yield of the Constant Maturity Long-Term Government of Canada Index and the Return on the Toronto Stock Exchange 300 Index - OLS Estimation

<table>
<thead>
<tr>
<th>Index</th>
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<th>$b$</th>
<th>$c$</th>
<th>$R^2$</th>
</tr>
</thead>
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<td>(0.24)</td>
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<td>-0.7903</td>
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<td></td>
<td>(0.41)</td>
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<tr>
<td></td>
<td>(0.53)</td>
<td>(-7.77)</td>
<td>(-3.22)</td>
<td></td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Index</th>
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<th>$b$</th>
<th>$c$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>(-3.02)</td>
<td>(-2.22)</td>
<td></td>
</tr>
<tr>
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<td>-1.2557</td>
<td>0.11</td>
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<tr>
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<td>(0.10)</td>
<td>(0.44)</td>
<td>(-2.87)</td>
<td></td>
</tr>
</tbody>
</table>

Table III reports the results of the OLS estimation of regression model (2) in which the dependent variable is the monthly change in the relative yield spread. This regression model is of the following form:

$$\Delta R = a + b \Delta PY + c I + \varepsilon,$$

where $\Delta R$ is the monthly change in the relative yield spread, $\Delta PY$ is the monthly percentage change in the yield of the constant maturity, long-term Government of Canada index, and $I$ is the monthly return on the Toronto Stock Exchange 300 index. $t$-values are in parentheses. Panel A reports the estimates for the entire sample, covering the 08:1976-07:2001 25-year period. Data during this sample period are dominated by corporate bonds carrying a standard call provision. Panel B outlines the results for the 01:1995-07:2001 sub-period, in which bonds carrying the doomsday call are expected to dominate all indices.

<table>
<thead>
<tr>
<th>Index</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$R^2$</th>
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</thead>
<tbody>
<tr>
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<td>-0.0553</td>
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</tr>
<tr>
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<table>
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<tbody>
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</tr>
<tr>
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<td>(-3.04)</td>
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Table IV
Regressions of Changes in Absolute Yield Spreads of SCM Long-Term Corporate Bond Indices on Changes in Government of Canada Yields - AR-GARCH Estimation

Table IV reports the results of the AR-GARCH estimation of regression model (8) in which the dependent variable is the monthly change in the absolute yield spread. This regression model is of the following form:

\[ \Delta S = \beta_0 + \beta_1 \Delta Y_{T-bill} + \beta_2 \Delta \text{lope} + \epsilon, \]

where \( \Delta S \) is the monthly change in the absolute yield spread, \( \Delta Y_{T-bill} \) is the monthly change in the three-month Treasury Bill yield, and \( \Delta Y_{T-bill} \) is the monthly change in the spread between the constant maturity long-term Government of Canada index and the three-month Treasury bill yield. \( t \)-values are in parentheses, \( m \) gives the degree of the autoregressive process as determined by the stepwise autoregression method, \( p \) and \( q \) are the GARCH(\( p,q \)) parameters, Norm. Test gives the \( p \)-value for the normality test for detecting misspecification of the GARCH model, and finally LM gives the \( p \)-value for the Lagrange multiplier test. Panel A reports the estimates for the entire sample, covering the 08:1976-07:2001 25-year period. Data during this sample period are dominated by corporate bonds carrying a standard call provision. Panel B outlines the results for the 01:1995-07:2001 sub-period, in which bonds carrying the doomsday call are expected to dominate all indices.

<table>
<thead>
<tr>
<th>Index</th>
<th>Regression Coefficients</th>
<th>AR and GARCH Parameters</th>
<th>Goodness of Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_0 )</td>
<td>( \beta_1 )</td>
<td>( \beta_2 )</td>
</tr>
<tr>
<td>AAA</td>
<td>-0.0027 (0.51)</td>
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<td>-0.0806 (-4.25)</td>
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<tr>
<td>AA</td>
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<tr>
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<td>-0.1011 (-6.22)</td>
<td>-0.1514 (-12.56)</td>
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<td>BBB</td>
<td>0.0025 (0.18)</td>
<td>-0.1244 (-3.26)</td>
<td>-0.1996 (-4.72)</td>
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<table>
<thead>
<tr>
<th>Index</th>
<th>Regression Coefficients</th>
<th>AR and GARCH Parameters</th>
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<tr>
<td></td>
<td>( \beta_0 )</td>
<td>( \beta_1 )</td>
<td>( \beta_2 )</td>
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<tr>
<td>AAA</td>
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<td>-0.0037 (-0.15)</td>
<td>0.1361 (1.21)</td>
<td>0.0947 (0.87)</td>
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Table A.1
Regressions of Changes in Absolute Yield Spreads of SCM Long-Term Corporate Bond Indices on Changes in the Yield of the Constant Maturity Long-Term Government of Canada Index and the Return on the Toronto Stock Exchange 300 Index - AR-GARCH Estimation

Table A.1 reports the results of the AR-GARCH estimation of regression model (1) in which the dependent variable is the monthly change in the absolute yield spread. This regression model is of the following form:

$$\Delta S = a + b \Delta Y + cI + \epsilon,$$

where $\Delta S$ is the monthly change in the absolute yield spread, $\Delta Y$ is the monthly change in the yield of the constant maturity, long-term Government of Canada index, and $I$ is the monthly return on the Toronto Stock Exchange 300 index. $t$-values are in parentheses, $m$ gives the degree of the autoregressive process as determined by the stepwise autoregression method, $p$ and $q$ are the GARCH$(p,q)$ parameters, Norm. Test gives the $p$-value for the normality test for detecting misspecification of the GARCH model, and finally LM gives the $p$-value for the Lagrange multiplier test. Panel A reports the estimates for the entire sample, covering the 08:1976-07:2001 25-year period. Data during this sample period are dominated by corporate bonds carrying a standard call provision. Panel B outlines the results for the 01:1995-07:2001 sub-period, in which bonds carrying the doomsday call are expected to dominate all indices.


<table>
<thead>
<tr>
<th>Index</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$m$</th>
<th>$p$</th>
<th>$q$</th>
<th>Norm. Test</th>
<th>$R^2$</th>
<th>LM</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.0007</td>
<td>-0.0563</td>
<td>-0.0302</td>
<td>1, 2</td>
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<td>1</td>
<td>&lt;0.0001</td>
<td>0.19</td>
<td>0.0004</td>
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<td>(0.12)</td>
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</tr>
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<td>1</td>
<td>&lt;0.0001</td>
<td>0.18</td>
<td>0.0006</td>
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<td>1</td>
<td>&lt;0.0001</td>
<td>0.19</td>
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<tr>
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### Panel B: 01:1995-07:2001

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<th>$m$</th>
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<th>LM</th>
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Table A.2

Regressions of Changes in Relative Yield Spreads of SCM Long-Term Corporate Bond Indices on Percentage Changes in the Yield of the Constant Maturity Long-Term Government of Canada Index and the Return on the Toronto Stock Exchange 300 Index - AR-GARCH Estimation

Table A.2 reports the results of the AR-GARCH estimation of regression model (2) in which the dependent variable is the monthly change in the relative yield spread. This regression model is of the following form:

\[ \Delta R = a + b \Delta PY + c I + \epsilon, \]

where \( \Delta R \) is the monthly change in the relative yield spread, \( \Delta PY \) is the monthly percentage change in the yield of the constant maturity, long-term Government of Canada index, and \( I \) is the monthly return on the Toronto Stock Exchange 300 index.

\( t \)-values are in parentheses, \( m \) gives the degree of the autoregressive process as determined by the stepwise autoregression method, \( p \) and \( q \) are the GARCH\((p,q)\) parameters, Norm. Test gives the \( p \)-value for the normality test for detecting misspecification of the GARCH model, and finally LM gives the \( p \)-value for the Lagrange multiplier test. Panel A reports the estimates for the entire sample, covering the 08:1976-07:2001 25-year period. Data during this sample period are dominated by corporate bonds carrying a standard call provision. Panel B outlines the results for the 01:1995-07:2001 sub-period, in which bonds carrying the doomsday call are expected to dominate all indices.

<table>
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<th>Index</th>
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<th>AR and GARCH Parameters</th>
<th>Goodness of Fit</th>
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<td>( b )</td>
<td>( c )</td>
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<th>Regression Coefficients</th>
<th>AR and GARCH Parameters</th>
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<td>(0.24)</td>
<td>(-2.07)</td>
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Figure 1. Moneyness of the doomsday call provision. To determine the moneyness of the doomsday call provision, we compare the doomsday spread to the yield spread. Based on a sample of doomsday spreads of long bonds for each month during the 01:1993-12:1999 period, for each rating category, we plot the yield spread on the corresponding long-term index (S), the average doomsday spread (μ), the average doomsday spread plus one standard deviation (μ+σ), and the average doomsday spread plus two standard deviations (μ+2σ).
Figure 2. Distribution of Canadian Corporate Bonds Based on Callability (Percentages), 01:1993 - 12:1999. For each month during the sampled 7-year period we count the number of corporate bonds issued with a doomsday call provision, the number of bonds issued with a standard call provision, and the number of noncallable bonds, as reported in the *Financial Post* Corporate Bond Record. In the Figure we plot the proportion of each of the above categories calculated with respect to the total number of bonds within each month.
Endnotes

1 Merton’s (1974) model for the valuation of corporate bonds stands as a milestone for the option-pricing literature in this area. In general, models following his work improve his model by allowing for (i) default prior to maturity (e.g., Black and Cox (1976), and Kim, Ramaswamy, and Sundaresan (KRS) (1993)); (ii) cash flow (as well as asset) based defaults (e.g., KRS (1993)); (iii) stochastic interest rates (e.g., Chance (1990), KRS (1993), and Shimko, Tejima and van Deventer (STD) (1993)); (iv) correlation between the firm’s assets and the stochastic interest rate (e.g., KRS (1993) and STD (1993)); and finally (v) valuation of coupon rather than zero-coupon bonds (e.g., KRS (1993), and Leland and Toft (1996)). The model presented by Longstaff and Schwartz (1995) incorporates all the above features, and also allows for multiple issues of debt, and deviations from the strict priority rule for the distribution of the firm’s assets in default.

2 As Longstaff and Schwartz note, under risk-neutral valuation, an upward static shift in the riskless rate implies a higher risk-neutral drift rate for the process, which governs the value of the firm’s assets.

3 Their sample consists of monthly data of investment grade bonds from the Moody’s industrial, utility, and railroad corporate bond yield averages for the 1977-1992 period. For each sector, and each investment grade, an average yield and average maturity are completed. Yield spreads are obtained by subtracting the average of 10-year and 30-year Treasury bonds (such that average maturities are matched). Note that in the empirical section of their paper, Longstaff and Schwartz use the term credit spreads for yield spreads. Strictly speaking, the credit spread of a callable corporate bond refers to the default premium required by bondholders, while the yield spread refers to the credit spread plus premiums related to both callability and liquidity.

4 Another implication of Longstaff and Schwartz’s (1995) model is that credit spreads are related to the correlation between firm value and interest rate. Their empirical results support this prediction as well. For comparable values of \( c \) across utility, industrial and railroad bonds (which tend to have a different correlation between assets and interest rate), the variation in \( b \) corresponds to variation in the correlation coefficient.

5 In our empirical analysis below, we use data for which constraints are imposed on the
level of coupon rates for corporate bonds, designed to eliminate the coupon level effect. Thus, in our estimation of regression model (1) for noncallable bonds, we do not find a significant absolute yield spread - riskless rate relation.

6 The reader may refer to Jacoby and Roberts (2001) for a detailed description of the SCM indices.

7 Note that since our sample is not stratified into different sectors, there is no need in the current study to use sector-specific stock indices as in Longstaff and Schwartz (1995).

8 Note that there is still a small probability that a BBB-rated bond will be upgraded in the future and that its doomsday call will be in-the-money. However, given the small rate of upgrades, the expected value of this state is trivial.

9 In a small number of cases, the cross-sectional standard deviation of the doomsday spread is zero, which implies: $\mu = \mu + \sigma = \mu + 2\sigma$. This is due to a small number of bonds available during the given month, usually issued by the same company, all sharing the same doomsday spread. In other cases, specifically in the 06:1993-07:1994 period, the reported doomsday spread for BBB-rated bonds is zero.

10 According to Duffee, following a given rise in Treasury yields, everything else being equal, the yield on a corporate bond will have to increase by a higher rate, so that the after-tax yield spread will remain unchanged. This implies that the pre-tax yield spread will widen following an increase in Treasury yields.

11 Since both tests are in agreement in all cases, for the sake of brevity we only report the results for the LM test below.

12 Gallant and Goebel (1976) describe the Yule-Walker method in detail.

13 A negative sign attributed to the default term in equation (6) can also contribute to a negative sign for the estimate of $b$. Below, we reject this hypothesis using the BBB index during the 01:1995-07:2001 period.

14 Note that the AR-GARCH(1,1) estimate of $c$ for the AAA index is insignificant. This result is expected, since the probability of default for AAA bonds is close to zero, which means that the default option for AAA bonds is deep out of the money. Thus, the asset factor for these bonds, estimated by $c$, becomes trivial.

15 The estimated coefficient $b$ for the BBB index under the Yule-Walker estimation is also
statistically insignificant. Thus, this result is robust.

16 A higher probability of a call for corporate bonds carrying the doomsday call provision reduces their effective duration, or price sensitivity to changes in the riskless rate. This implies that following an upward shift in the riskless rate, the corporate bond yield will rise by a lower rate, and yield spreads will contract.

17 Note that we obtain the same result for the BBB index when the Yule-Walker procedure is applied.

18 In analyzing the results, note that one’s conclusions are insensitive to whether one uses OLS or the estimation method we apply here.