The Impact of Investment Strategy of DC Pension Plan on Retirement Age Distribution

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Investment Model and Assumptions

Single-Asset and Mixed-Asset Portfolios

The Effect of Switch Decisions

DC Pension System Imposed On The Whole Population
Investment Model

- State-space model for the log growth of CPI and short rate
- Term premium is influenced by the change of short rate.
- Positive change in short rate could have negative effect on the Stock
Simulated Paths

**Short rate and Long rate**

**Stock Index**

**Consumer Price Index**

**Wage Index**
Other Assumptions

Uniform Retirement Decision

- Entry Age = 25
- Salary Growth

\[
R(t) = \frac{m(t)}{m(0)} \times \text{Wage-index}(t)
\]

- Wealth Accumulation

\[
W(e, t) = [W(e, t-1) + 0.1 \times R(e, t-1)] \times (1 + i(e + t))
\]

- 2% Indexed-Annuity.

\[
\ddot{a}(e, t, e_a) = \sum_{s=0}^{\omega-e_a-t} 1.02^s \times (1 + i_{e+t})^{-s} \cdot s p_{e_a+t}
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Other Assumptions: Cont’d

- Replacement Ratio (RR) and Retirement Decision.

\[
RR(e, t) = \frac{Wealth(e, t)/\bar{a}_{e+t}}{salary(e, t)}
\]

\[
ERA = \min\{e_a + t : RR(e, t, e_a) \geq MRR(e_a + t)\}
\]
Investment Model and Assumptions  Single-Asset and Mixed-Asset Portfolios  The Effect of Switch Decisions  DC Pension System Imposed On The Whole Population

- Static Population Structure
- Dependency Ratio

\[
\text{Dependency Ratio} = \frac{\text{Number of retirees}}{\text{Number of working people}}
\]
Single-Asset Portfolios

Simulated Empirical CDF of Retirement Age
For Each Single-Asset Portfolio
Mixed-Asset Portfolios And Efficiency Portfolios

Simulated Efficient Portfolio for Fixed-Allocation Portfolios

Tangency Portfolio: 20% stock 80% bond
The Effect of Switch Decisions

- Why switch? Supposed people all enter the plan at age 25.
The Effect of Switch Decisions: Cont’d

the Switch decision is examined every other year since entry

The Effect of Switch From Portfolio C to A at Different Ages

![Graph showing the effect of switching from Portfolio C to A at different ages. The graph plots mean against standard deviation for ages 26 to 42 with a separate line for ages 42 to 60. The graph illustrates how switching impacts the mean and standard deviation over time.]
Categories:

- Entry age: 25 or 30
- Portfolio A, B, C, A→ B, A→ C, or B→ C
- Switch Timing: age 35 or age 45

18 categories in total. Each category assumes equal weights. Then what happened?
Simulated Series of Worker–To–Beneficiary Ratio For Five Hundred Years

Worker to Beneficiary Ratio

Equivalent Ratio under Defined Benefit Plan
Thanks for your attention. Question Time