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1 We gratefully acknowledge financial support from the Society of Actuaries (CAE grant)
Introduction

1. Risk-Neutral Valuation from Policyholder’s Perspective
2. Empirical Analysis of Prudential’s ASL II
3. Implications for Product Design: Neg. Option Values
4. Conclusions
Introduction
Motivation

- Exercise-dependent features in personal savings products increasingly popular
  - Surrender options
    - Grosen and Jørgensen, IME 2000
    - Zaglauer and Bauer, IME 2008
  - Withdrawal guarantees in Variable Annuities (VA)
    - Milevsky and Salisbury, IME 2006
    - Dai et al., MathFin 2007
    - Bauer et al., ASTIN 2008
  - GMWB for Life
    - Steinorth and Mitchell, 2011
  - Real option to transfer between subaccounts
    - Ulm, JRI 2006

- Policyholder behavior can affect valuation tremendously
  - Kling et al., 2011
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Actuarial literature: optimal exercise based on arbitrage-pricing

Inconsistent with empirically observed patterns and prices

Reasons: arbitrage pricing assumptions may be violated

Life insurance market incomplete

Cannot sell – or repurchase – policies at risk-neutral value

Withdrawing means possibly giving up guarantees and other benefits

Market frictions

Taxation differs (1) between policyholder and company, and (2) across policyholder’s investment options

Ex.: Variable Annuities grow tax-deferred

Consider poster-child of exercise-dependent features

Withdrawal guarantees in Variable Annuities

But: General methodology applies to various personalized savings products with guarantees
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- Reasons: arbitrage pricing assumptions may be violated
  - Life insurance market incomplete
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• Consider poster-child of exercise-dependent features
  ➞ Withdrawal guarantees in Variable Annuities

• But: General methodology applies to various personalized savings products with guarantees
Recent troubles for insurers in US Variable Annuity market:

- *The Hartford* accepted $3.4B in TARP money
- *ING USA* downgraded to “A” after announcing $1.1B earnings charge against VAs
- Many insurers increased guarantee fees or dropped out of VA market

Similar problems in Japan

- Sumitomo Life forced to increase capital stock on Japanese VA portfolio
- Several large insurers withdrew from Japanese VA market

“The problem of the current (Japanese) VA market is not lack of demand, but lack of supply from willing insurers.” (Watson Wyatt)
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“The problem of the current (Japanese) VA market is not lack of demand, but lack of supply from willing insurers.” (Watson Wyatt)
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- **Variable Annuities (VA)**
  - Popular long-term investment option (in U.S.)
  - Grow tax-deferred
  - Investment in stock portfolio / mutual fund
  - Risky payout profile
  - Guaranteed Minimum Benefits available for downside protection

- **Guaranteed Minimum Withdrawal Benefit (GMWB)**
  - Option to withdraw initial investment over time, regardless of account performance
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• Simple example:
  - Policyholder invests $100K in mutual fund with insurer for 15 years
  - Right (but not obligation) to withdraw $7K each year
  - Until $100K have been withdrawn on aggregate
  - Can withdraw more than $7K, only if account value permits (and fee may apply)
  - Pay 50 bps of account value in guarantee fees annually
  - At death: bequestors receive account value
  - If alive at maturity: receive account value

• Previous literature, based on RNV: Optimal to
  - Keep withdrawing guaranteed amount
  - Surrender when VA account value large, to avoid guarantee fees
  - Derived prices significantly above market rates

• This is not what we find!
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Thorsten Moenig
Revisiting the Risk-Neutral Approach to Optimal Policyholder Behavior
Motivation

• Life-cycle model to address market incompleteness
  ▶ Results driven by “Subjective” Value Maximization
    ★ Taxation matters
  ▶ See Moenig and Bauer (2011), presented at ARC 46

• This paper: Risk-neutral valuation from policyholder’s perspective
  ▶ Develop valuation framework with differing tax schemes
  ▶ Apply to VA + GMWB contracts
    ★ Consideration of taxes alone explains concurrent market rates
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1 Introduction

2 Risk-Neutral Valuation from Policyholder’s Perspective
   Valuation of Cash-Flows with Differing Taxation Schemes
   Optimal Withdrawal Behavior

3 Empirical Analysis of Prudential’s ASL II

4 Implications for Product Design: Neg. Option Values

5 Conclusions
Cash-flow taxed differently than replicating portfolio

- Ross, JPE 1986: No universal pricing measure
- Valuation of cash-flows locally (i.e. agent-specific / subjective)

I develop subjective valuation approach, allowing for different assets with differing tax treatments

- Assume complete pre-tax market
- Determine time-$t$ value ($X_t$) of post-tax cash flow $X_{t+1}$
  - Define $X_t$ as amount needed to attain $X_{t+1}$, after taxes
  - Calculate pre-tax amount at time $t+1$ that yields $X_{t+1}$ after taxes
  - “Discount” to time $t$ with (unique) pre-tax measure $Q$

**Proposition 1.**

Any post-tax cash flow $X_{t+1}$ can be valued uniquely at time $t$ as

$$X_t = \mathbb{E}_t^Q \left[ \frac{B_t}{B_{t+1}} \cdot (X_{t+1}) \right] + \frac{\kappa}{1-\kappa} \cdot \mathbb{E}_t^Q \left[ \frac{B_t}{B_{t+1}} \cdot (X_{t+1} - X_t)^+ \right]. \quad (1)$$
Risk-Neutral Valuation from Policyholder’s Perspective

Valuation of Cash-Flows with Differing Taxation Schemes

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Optimal Withdrawals: \( t = 10, \ H_t = G_t = 100. \) (in 1000)

\[ w_t^* = \max(\text{Guarantee}, X_t^-) \]

- \( w_t^* \) No Taxes
- \( w_t \)
- \( w_t \) Taxes
- \( \max(\text{Guarantee}, X_t^-) \)
Accounting for taxation has tremendous impact

<table>
<thead>
<tr>
<th></th>
<th>With Taxes</th>
<th>W/o Taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^Q[\text{Fees}]$</td>
<td>5,708</td>
<td>3,299</td>
</tr>
<tr>
<td>$E^Q[\text{Excess-Fee}]$</td>
<td>162</td>
<td>10</td>
</tr>
<tr>
<td>$E^Q[\text{GMWB}]$</td>
<td>2,094</td>
<td>3,163</td>
</tr>
<tr>
<td>$E[\text{agg. w/d}]$</td>
<td>19,240</td>
<td>191,320</td>
</tr>
<tr>
<td>$P(G_T = 0)$</td>
<td>9.3%</td>
<td>83.6%</td>
</tr>
<tr>
<td>$P(G_T &lt; P_0)$</td>
<td>13.0%</td>
<td>88.7%</td>
</tr>
</tbody>
</table>
Empirical Analysis of Prudential’s ASL II

1. Introduction
2. Risk-Neutral Valuation from Policyholder’s Perspective
3. Empirical Analysis of Prudential’s ASL II
   - Product Description
   - Results
4. Implications for Product Design: Neg. Option Values
5. Conclusions
Empirical Analysis of Prudential’s ASL II

Product Description

- Implement VA offered in U.S. market
  - *ASL II by Prudential Annuities Life Assurance Corporation*

- Key differences to simple GMWB example
  - Charges of 165 bps (of account value) p.a. (for M&E risk and Admin.)
  - Basic death benefit included
  - GMWB eligible for additional 35 bps p.a.
    - Includes step-up option
    - At maturity or death of PH: option to receive remaining benefits base, annuitized with zero interest
    - Guarantee fee waived after 7 years, if no withdrawals are made
  - Investment in riskiest eligible fund: *Pro Fund VP Bull*
    - Returns similar to *S&P500*

- Implement optimization with subjective RNV approach
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### Empirical Analysis of Prudential’s ASL II

#### Results

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>$\sigma = 20%$</th>
<th>$r = 3%$</th>
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<th>$\kappa = 20%$</th>
<th>No Taxes</th>
</tr>
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<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$E^Q[\text{Guarantee}]$</td>
<td>4,161</td>
<td>9,992</td>
<td>16,866</td>
<td>22,060</td>
<td>7,768</td>
</tr>
<tr>
<td>$E^Q[\text{Fees}]$</td>
<td>11,140</td>
<td>19,692</td>
<td>22,480</td>
<td>23,809</td>
<td>22,379</td>
</tr>
<tr>
<td>$E^Q[\text{Net Profit}]$</td>
<td>6,979</td>
<td>9,700</td>
<td>5,614</td>
<td>1,748</td>
<td>14,611</td>
</tr>
<tr>
<td>Surrender Rate</td>
<td>75.2%</td>
<td>37.0%</td>
<td>19.4%</td>
<td>20.1%</td>
<td>21.0%</td>
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|                 | $\kappa = 20\%$ | No Taxes |
|                 |                 |          |
| Without GMWB    |                 |          |
| $E^Q[\text{Guarantee}]$ | 799            | 1,202    | 1,870     | 2,535           | 922      | 41       |
| $E^Q[\text{Fees}]$     | 8,579           | 8,858    | 9,485     | 9,702           | 12,884   | 1,636    |
| $E^Q[\text{Net Profit}]$| 7,780           | 7,657    | 7,615     | 7,167           | 11,962   | 1,596    |
| Surrender Rate   | 88.2%           | 84.5%    | 79.2%     | 74.1%           | 75.1%    | 99.2%    |
| $E^Q[\text{GMWB}]$  | -802            | 2,044    | -2,001    | -5,419          | 2,648    | 206      |
Empirical Analysis of Prudential’s ASL II

Results

• Insurer collects decent surplus in both cases
  ▶ Benchmark case: ca. 7% of initial investment
  ▶ Might be used to cover administrative costs and other expenses

• 35 bps roughly fair price for GMWB
  ▶ Results sensitive to financial market parameters

• Significant loss when interest rates low and volatility high
  ▶ Might explain modifications of GMWBs and discontinuation of many VA products in recent years

• Without taxes: PH surrenders almost immediately
  ▶ Why invest in VA in the first place?

• High surrender rate
  ▶ PH withdraws when guarantee out of money
  ▶ Tax-deferred growth not generally worth 165 bps

• Also: time to maturity and income tax rate matter little
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4. Implications for Product Design: Neg. Option Values
   - Mechanics
   - Description
   - Results

5. Conclusions
Implications for Product Design: Neg. Option Values

- Conventional wisdom: options have non-negative value
  - Option holder cannot be worse off than without the option
  - Issuer responsible for payout when option is exercised
- Assumes both parties have identical value functions
- Not true in many personal savings products
  - Incomplete market
  - Preferential tax treatment of underlying investments
- Investor’s optimal exercise strategy no longer worst-case for issuer
  - Decisions affected by preferences and/or taxation
    ⇒ True even if investor is value maximizer
- Negative option values become possible
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- Suppose presence of one option affects exercise of other (explicit or implicit) options
  - Marginal value of option may depend on investor’s portfolio
  - Value can be much smaller than option payout
  - Combined options cheaper than sum of individual option prices
  - See e.g. Bauer et al., ASTIN 2008

- True even under arbitrage pricing
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- Taxation introduces third party: government
  - Third party cannot affect exercise behavior directly
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Implications for Product Design: Neg. Option Values

Mechanics

- Option may induce exercise strategy with lower overall tax payments
  - Investor gains ✓
  - Government “loses” ✓
  - Issuer: ??

- In extreme (but possible) cases: issuer better off with writing the option
  - Both issuer and investor benefit from option
  - At financial expense of government
  - Option has negative marginal value to its issuer

- Example: Death benefit guarantee (GMDB) in Variable Annuity, when GMWB is present
  - Standard feature in most VA products
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Implications for Product Design: Neg. Option Values

Description

- Demonstrate possibility of negative option values in two-period model
- Also in practice: Implement (slightly modified version of) Prudential’s ASL
  //
  - Includes GMWB, but no maturity benefits
- Methodology and parameter specifications from Essay 1
  - VA charges of 165 bps (of account value) p.a.
  - Plus 35 bps p.a. for GMWB, while applicable
Implications for Product Design: Neg. Option Values

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\[
\begin{array}{cc|cc}
 & \text{With GMDB} & \text{Without GMDB} \\
\hline
\mathbb{E}^\mathbb{Q}[\text{Guarantee}] & 3,610 & 3,340 \\
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2 Risk-Neutral Valuation from Policyholder’s Perspective

3 Empirical Analysis of Prudential’s ASL II

4 Implications for Product Design: Neg. Option Values

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  - (Novel) Arbitrage pricing approach
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  - Applied to (simplified) sample contract as well as empirical VA
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  - Both insurer and investor better off with guarantee
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