ACT 3130
Test 2
Feb 25, 2016

Name:
Student ID:

Closed notes and books. You are allowed to use a calculator. To get full credit, be sure to show all your work on the questions.

1. A 3-year term endowment insurance on (30) pays \$1000 at the end of the year of death. The endowment benefit is \$1000.

   You are given $\delta = 0.05129329$, $q_{30} = 0.15$, $q_{31} = 0.2$. Calculate the net single premium.

   [5 marks]

2. A 20-year term endowment insurance on (40) pays \$1000 at the end of the year of death. The endowment benefit is \$300.

   You are given $i = 0.06$ and mortality follows the Illustrative Life Table.

   Z is the present value random variable of this insurance. Calculate $E(Z^2)$.

[5 marks]

3. A whole life insurance package is sold to a company to insure 100 employees: 70 of them are forty years old, and the rest are fifty years old. Future lifetimes are independent.

   Death benefits are \$100 payable at the end of the year of death. You are given $i = 0.06$ and mortality follows the Illustrative Life Table.

   Using normal approximation, calculate the required funds F such that the probability the insurer will be able to pay all of the claims is 0.95.

   [5 marks]

4. A whole life insurance on (x) pays a death benefit **at the moment of death**. You are given

   (i) $\mu_{x+t} = \frac{3130^{3230}}{\omega - t}$ for $0 \leq t \leq \omega$.

   (ii) $\delta = 0.03179$

   (iii) $b_t = \$3130(1.03230)^t$ for $t > 0$.

   Calculate the net single premium.

[5 marks]

5. You are given $\delta = \begin{cases} 0.06 & t \leq 10 \\ 0.07 & t > 10 \end{cases}$, and $\mu_{30+t} = \begin{cases} 0.03 & t \leq 10 \\ 0.04 & t > 10 \end{cases}$

Calculate $\bar{A}_{30:\overline{20|}}$

[5 marks]

6. For a whole life insurance on (30), the death benefits are \$1000 for the first 5 years, \$2000 for the second 5 years, and \$3000 thereafter payable at the moment of death.

   You are given ${}_tq_{30} = \frac{t}{70}$ for $0 \leq t \leq 70$, and $\delta_t = 0.06$.

   Z is the present value random variable of this insurance. Calculate $Pr(Z > 1200)$.

   [5 marks]

7. If deaths are uniformly distributed between integral ages, then we know that $\delta \bar{A}_x = i A_x$.

   Show that or explain why ${}^2\bar{A}_x = c\ {}^2A_x$, where $c = \frac{2i - i^2}{2\delta}$.

[5 marks]

1. A 3-year term endowment insurance on (x) pays \$1000 at the end of the year of death. The endowment benefit is \$300.

   You are given $\delta_t = \mu_t = 0.05129329$ for $t \geq 0$. Calculate the net single premium.

   [5 marks]

$$
\begin{aligned}
\nu &= e^{-0.05129329} \\
&= 0.95 \\
{}_kp_x &= e^{-0.05129329k} \\
&= 0.95^k \\
q_{x+k} &= 1 - p_{x+k} \\
&= 1 - e^{-0.05129329} \\
&= 1 - 0.95 \\
&= 0.05
\end{aligned}
$$

$$
\begin{aligned}
EPV &= \sum_{k=0}^{2} 1000\ \nu^k\, {}_kp_x q_{x+k} \ +\ 300\ \nu^3\, {}_3p_x \\
&= 1000(\ 0.95(0.05) + 0.95^2(0.95)(0.05) + 0.95^3(0.95)^2(0.05)) + 300(0.95^3(0.95^3)\ ) \\
&= 349.58
\end{aligned}
$$

2. A 20-year term endowment insurance on (40) pays \$1000 at the end of the year of death. The endowment benefit is \$300.

   You are given $i = 0.06$ and mortality follows the Illustrative Life Table.

   Z is the present value random variable of this insurance. Calculate $E(Z^2)$.

   [5 marks]

   From the ILT, ${}^2A_{40} = 0.04863$, ${}^2A_{60} = 0.17741$, ${}_{20}E_{40} = 0.27414$.

$$
\begin{aligned}
{}^2A_{40} &= {}^2A_{\substack{1\\40:\overline{20|}}} + (\nu^{20})^2\ {}_{20}p_{40}\ {}^2A_{60} \\
{}^2A_{40} &= {}^2A_{\substack{1\\40:\overline{20|}}} + \nu^{20}\ {}_{20}E_{40}\ {}^2A_{60} \\
0.04863 &= {}^2A_{\substack{1\\40:\overline{20|}}} + (\frac{1}{1.06})^{20}\ (0.27414)\ (0.17741) \\
{}^2A_{\substack{1\\40:\overline{20|}}} &= 0.03347
\end{aligned}
$$

$$
\begin{aligned}
E(Z^2) &= 1000^2\ {}^2A_{\substack{1\\40:\overline{20|}}} + 300^2\ (\nu^{20})^2\ {}_{20}p_{40} \\
&= 1000^2\ 0.03347 + 300^2\ (\frac{1}{1.06})^{20}\ (0.27414) \\
&= 41158.36
\end{aligned}
$$

3. A whole life insurance package is sold to a company to insure 100 employees: 70 of them are forty years old, and the rest are fifty years old.

   Death benefits are \$100 payable at the end of the year of death. You are given $i = 0.06$ and mortality follows the Illustrative Life Table.

   Using the normal approximation, calculate the required funds F such that the probability the insurer will be able to pay all of the claims is 0.95.

   [5 marks]

   From the ILT, $A_{40} = 0.16132$, ${}^2A_{40} = 0.04863$, $A_{50} = 0.24905$, ${}^2A_{50} = 0.09476$.

   $S = 70\ Z_{40} + 30\ Z_{50} \dot{\sim} N(\ E(S),\ Var(S)\ )$

   $E(S) = 70\ (\$100)\ A_{40}\ +\ 30\ (\$100)\ A_{50}\ =\ \$1876.39$

   $Var(S) = 70\ (\$100^2)\ ({}^2A_{40} - {A_{40}}^2)\ +\ 30\ (\$100^2)\ ({}^2A_{50} - {A_{50}}^2)\ =\ \$25644.33$

$$
\begin{aligned}
Pr(S < F) &= 0.95 \\
\phi(\frac{F - 1876.39}{\sqrt{25644.33}}) &= 0.95 \\
\frac{F - 1876.39}{\sqrt{25644.33}} &= 1.645 \\
F &= 2139.82
\end{aligned}
$$

4. A whole life insurance on (x) pays a death benefit at the moment of death. You are given

   (i) $\mu_{x+t} = \frac{3130^{3230}}{\omega - t}$ for $0 \leq t \leq \omega$.

   (ii) $i = 0.04$

   (iii) The death benefit, starting at \$1000, is continuously increased by a rate of 4% per year.

   Calculate the net single premium.

[5 marks]

$\nu^t = (\frac{1}{1.04})^t$

Death benefit = \$1000 $(1.04)^t$

$$\begin{aligned} EPV &= \int_0^\omega 1000\ (1.04)^t\ (\frac{1}{1.04})^t\ f_x(t)\ dt \\ &= \int_0^\omega 1000\ f_x(t)\ dt \\ &= 1000\ (1) \\ &= 1000 \end{aligned}$$

5. You are given $\delta = \begin{cases} 0.06 & t \leq 10 \\ 0.07 & t > 10 \end{cases}$, and $\mu_{30+t} = \begin{cases} 0.03 & t \leq 10 \\ 0.04 & t > 10 \end{cases}$

Calculate $\bar{A}_{30:\overline{20|}}$

[5 marks]

$$\begin{aligned} \bar{A}_{30:\overline{20|}} &= \bar{A}^{\ 1}_{30:\overline{20|}} + \bar{A}^{\ \ \ 1}_{30:\overline{20|}} \\ &= \bar{A}^{\ 1}_{30:\overline{10|}} + {}_{10}E_{30}\ \bar{A}^{\ 1}_{40:\overline{10|}} + \bar{A}^{\ \ \ 1}_{30:\overline{20|}} \\ &= 0.43178 \end{aligned}$$

where

$\bar{A}^{\ 1}_{30:\overline{10|}} = \int_0^{10}\ e^{-0.06t}\ e^{-0.03t}\ 0.03\ dt = 0.19178$

${}_{10}E_{30} = e^{-0.06(10)}\ e^{-0.03(10)} = 0.4066$

$\bar{A}^{\ 1}_{40:\overline{10|}} = \int_0^{10}\ e^{-0.07t}\ e^{-0.04t}\ 0.04\ dt = 0.24259$

$\bar{A}^{\ \ \ 1}_{30:\overline{20|}} = e^{-0.06(10)-0.07(10)}\ e^{-0.03(10)-0.04(10)} = 0.13536$

6. For a whole life insurance on (30), the death benefits are \$1000 for the first 5 years, \$2000 for the second 5 years, and \$3000 thereafter payable at the moment of death.

You are given ${}_tP_{30} = \frac{70-t}{70}$ for $0 \le t \le 70$, and $\delta_t = 0.06$.

Z is the present value random variable of this insurance. Calculate $Pr(Z > 1200)$.

[5 marks]

$$Z = \begin{cases} 1000e^{-0.06t} & 0 \le t < 5 \\ 2000e^{-0.06t} & 5 \le t < 10 \\ 3000e^{-0.06t} & 10 \le t < 70 \end{cases}$$

$$\begin{aligned}
Pr(Z > 1500 \mid 0 \le t < 5) &= Pr(1000e^{-0.06t} > 1200 \mid 0 \le t < 5) \\
&= Pr(t < \frac{ln1.2}{-0.06} \mid 0 \le t < 5) \\
&= Pr(t < -3.04 \mid 0 \le t < 5) \\
&= 0 \\
Pr(Z > 1500 \mid 5 \le t < 10) &= Pr(2000e^{-0.06t} > 1200 \mid 5 \le t < 10) \\
&= Pr(t < \frac{ln0.6}{-0.06} \mid 5 \le t < 10) \\
&= Pr(t < 8.51 \mid 5 \le t < 10) \\
&= \frac{Pr(5 \le t < 8.51)}{Pr(5 \le t < 10)} \\
&= \frac{8.51 - 5}{10 - 5} \\
&= 0.7028 \\
Pr(Z > 1500 \mid 10 \le t < 70) &= Pr(3000e^{-0.06t} > 1200 \mid 10 \le t < 70) \\
&= Pr(t < \frac{ln0.4}{-0.06} \mid 10 \le t < 70) \\
&= Pr(t < 15.27 \mid 10 \le t < 70) \\
&= \frac{Pr(10 \le t < 15.27)}{Pr(10 \le t < 70)} \\
&= \frac{15.27 - 10}{70 - 10} \\
&= 0.0879 \\
Pr(Z > 1500) &= \frac{5-0}{70}(0) + \frac{10-5}{70}(0.7082) + \frac{70-10}{70}(0.0879) \\
&= 0.1255
\end{aligned}$$

7. If deaths are uniformly distributed between integral ages, then we know that $\delta \bar{A}_x = i A_x$.

Show that or explain why ${}^2\bar{A}_x = c\ {}^2A_x$, where $c = \frac{2i - i^2}{2\delta}$.

[5 marks]

When we add a "2" in front of $\bar{A}_x$ or A_x, it means that the rate of discount ν is squared.

Hence similarly, ${}^2\delta\ {}^2\bar{A}_x = {}^2i\ {}^2A_x$

For the continuous case, $\nu = e^{-\delta}$, and $\nu^2 = e^{-2\delta} = e^{-{}^2\delta}$. Therefore, ${}^2\delta = 2\delta$.

For the discrete case, $\nu = \frac{1}{1+i}$, and $\nu^2 = (\frac{1}{1+i})^2 = \frac{1}{1+{}^2i}$. So, $(1+i)^2 = 1 + {}^2i$ and ${}^2i = 2i - i^2$.

So we have $2\delta\ {}^2\bar{A}_x = (2i - i^2)\ {}^2A_x$.